OPTIMAL REGULATORS
FOR SYNCHRONOUS MACHINES

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"To My Beloved Parents"

CERTIFICATE

Certified that this work, "Optimal Regulators for Synchronous Machines" by Mr. M. Arumugam has been carried out under my supervision and that it has not been submitted elsewhere for a degree.

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NOMENCLATURE

X	-	n-dimensional state vector
Y	-	m-dimensional output vector
u	-	r-dimensional control vector
Z	-	p-dimensional observer state vector
A, A_1	-	system matrices
B	-	system input matrix
C	-	output matrix
P, K	-	Riccati matrices
Q, R	-	weightage matrices
F,G,H,T,L,W	-	observer matrices of appropriate dimensions
S	-	Schwarz canonical matrix
J	-	performance index
$\hat{J}$	-	average value of performance index
b,f,g	-	column vectors
M	-	inertia constant of machine
K_d	-	damping coefficient of machine
d.w.r.	-	divided winding rotor
tr(.)	-	trace of the matrix (.)
E(.)	-	expected value of (.)
p	-	time derivative operator
t	-	time in seconds
Δ	-	incremental operator
d	-	direct axis of the machine
q	-	quadrature axis of the machine
dia(.)	-	diagonal matrix (.)

w - instantaneous angular frequency, rad./sec.

w_o - rated angular frequency, rad/sec.

δ - rotor angle with respect to the infinite bus

V_o - infinite bus voltage

V_m - machine terminal voltage

V_i - internal voltage of machine

v_d, v_q - direct and quadrature axis voltages

i_d, i_q - direct and quadrature axis currents

v_{fd} - direct axis field voltage (normal machine)

E_{fd} - $v_{fd}\ x_{md}/r_{fd}$, field voltage referred to stator

i_{fd} - direct axis field current (normal machine)

v_t, v_r - torque and reactive winding voltages

i_t, i_r - torque and reactive winding currents

x_d - d-axis synchronous reactance

x_q - q-axis synchronous reactance

x_{ad}, x_{aq} - mutual reactances in d and q axis circuits

x_t, x_r - total reactance of torque and reactive windings

x_{tr} - mutual reactance between torque and reactive windings

x_{at}, x_{ar} - mutual reactances of armature with torque and reactive windings

x_{atd}, x_{ard} - mutual reactances of torque and reactive windings with d-axis circuit

x_{atq}, x_{arq} - mutual reactances of torque and reactive windings with q-axis circuit

$x_{atd} = x_{ard} = x_{ad} \cos 30$

$x_{atq} = x_{arq} = x_{aq} \sin 30$

x_{kkd}, x_{kkq} – total reactances of d and q axis damper bar circuits

r_a – armature circuit resistance

r_{fd} – direct axis field winding resistance

r_t, r_r – torque and reactive winding resistances

r_{kd}, r_{kq} – d and q axis damper bar circuit resistances

ψ_{fd} – direct axis field flux linkage

ψ_d – flux linkage with d-axis winding

ψ_{kd} – d-axis damper bar flux linkage

ψ_q – flux linkage of q-axis winding

ψ_{kq} – q-axis damper bar flux linkage

ψ_t – torque winding flux linkage

ψ_r – reactive winding flux linkage

r_e – transmission line resistance

x_e – transmission line reactance

K_v, T_v – voltage regulator gain and time constant

K_s, T_s – stabilizer gain and time constant

K_a, T_a – angle regulator gain and time constant

K_g – speed governor gain

T_g, T_h – time constants of governor and turbine

Subscripts –

d – d-axis quantities

q – q-axis quantities

o – operating point quantities

ref – reference quantity

Superscript –

* – variables in the reduced system

SYNOPSIS

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OPTIMAL REGULATORS FOR SYNCHRONOUS MACHINES

In recent times, the attention of power system Engineers has been focussed on methods of increasing transient and dynamic performance of power systems in order to minimize or eliminate the effects of severe system oscillations, using modern control theoretic concepts. By regulating the terminal voltage and speed to some fixed reference, it has been shown that the transient performance of the synchronous generators can be improved.

The conventional design procedure for such voltage regulators and speed governors assumes apriori knowledge of suitable configurations for the regulating and stabilizing equipments. This has drawbacks in the arbitrary choice of the controller configurations and the cut and try procedure involved in the selection of regulator parameters to meet the design specifications such as overshoot, steady state error and stability limit. Even when these specifications are met, it may not be the best possible design. The notion of modern

and optimal control theory provides a convenient framework for incorporating the design specifications and for studying the system dynamic behaviour. Through the proper choice of an appropriate performance criterion, it becomes possible to impart the desired features to the system transient and dynamic behaviour.

The objective of the present thesis is to formulate the synchronous machine control as an optimal control problem and then to obtain an integrated control for the excitation and prime mover control, which is optimal with respect to a chosen performance criterion.

Chapter I is a general introduction giving a brief discussion on the synchronous machine control problem and the modern development with particular reference to the present status of optimal control of power system dynamics.

Chapter II is devoted to the development of a mathematical model for the synchronous machine connected to an infinite bus system. The state space model of the single machine system is derived using winding currents as some of the state variables instead of flux linkages, in a form most suitable for the application of optimal control theory. For small disturbances in the system which occur continuously during the normal operation,

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overall configuration of the controlled system. The transfer matrix relating the outputs to the inputs are also derived.

The observer designed in the last chapter introduces dynamics in the control loops and also exponentially decaying error. Hence it is desired to find a control law which is function of measurable outputs. Chapter VI deals with the suboptimal control of synchronous machine system. Here the regulator is constrained to be a linear time invariant function of the measurable system outputs. Using an iterative algorithm the feedback control matrix is determined. The effect of the suboptimal control law determined for a particular operating point when used at different load levels is discussed and conclusions are drawn.

The stability of a conventional synchronous machine delivering leading power factor loads is poor because of lower excitation. The problem of maintaining stability under these conditions by using a divided winding rotor synchronous machine is discussed in Chapter VII. A state space model is derived and the behaviour of the system with conventional angle and voltage regulators is investigated.

In Chapter VIII, the optimal and suboptimal control of the divided winding rotor synchronous machine is dealt with. The system behaviour with the optimal controller

for large disturbances is discussed. The superiority of optimal regulators over conventional regulators is established.

The disturbances occuring in a power system are random in nature and also uncertainties are introduced in the output measurements. Hence the design of optimal regulators should take into effect the presence of random disturbances. In Chapter IX the optimal control of synchronous machine in the presence of such random disturbances is described. The disturbances are assumed to be white noise with known statistics. The average behaviour of the system in the presence of noise is then determined. The optimal control law for these conditions is the same as that for the deterministic case, but the performance index value is increased on an average.

In response to the demands of the system growth and more interactions of control systems, the dynamic analysis becomes more expensive and time consuming even on a fast digital computer. The question might, therefore, be asked whether simplified models might not be appropriate for such systems. A method of simplifying such large linear dynamic systems using Schwarz canonical form which does not require the computation of eigen values and eigen vectors as hither to done in existing methods, is described in Chapter X. The response of the original and simplified models is compared.

The method of state variable grouping is used to obtain a reduced model for multivariable control systems. Using the reduced model, an optimal regulator is obtained. This control law is used as a suboptimal control law for the original system. The performance of the original system is investigated with this suboptimal control law.

In the concluding chapter, the results of the thesis are reviewed and future lines of research are delinated.

CHAPTER I

INTRODUCTION

1.1 SYNCHRONOUS MACHINE CONTROL

In the developing countries of the world, the use of electricity has been doubling every 10 years. The rapid and continuing expansion has led to the large extra high voltage interconnected power systems spreading across national boundaries. The growing demand for better security and continuity of service on one hand and investment and operating cost reduction on the other hand affect the role of controls both in system design and operation. Even though the security is the most important aspect of quality of service, the control systems play an important role in minimizing the system frequency and voltage deviations.

With advent of fast acting voltage regulators and significant development in control system theory, the study of stability of synchronous machine system provided with these regulators, called the dynamic stability, is assuming much importance. The synchronous machines provided with continuously acting automatic voltage regulators can be operated beyond their steady state stability limits. The machine falls out of step or becomes unstable when the dynamic stability limit is reached. The limit depends not only on the machine

characteristics but also upon the system load and regulator characteristics. Greater reliance is, therefore, placed on well designed control systems. Automatic control techniques are conceived as a means to obtain better operating performance for a given system and these are used towards improvement of overall system response by designing proper feedback loops. The concepts of modern control theory and methods from multivariable system matrix analysis and optimization have a very important bearing on the basic engineering approach to the system analysis for design and operation.

In the conventional methods of design and analysis of synchronous machine for control of voltage and frequency, the machines were represented by simple models. The control of voltage and frequency is viewed as two distinct control systems. Separate configurations are chosen with no interaction between them. The frequency is controlled by primemover governor which derives a feedback signal from the rotor speed and its derivative. More recently rotor angle control is also used. Similarly, the voltage regulator derives a signal proportional to the terminal voltage and/or some function of the same. Apriori configurations of the regulators for both frequency and voltage controls are assumed independently.

The complete model including the machine dynamics and regulators are derived as a single higher order differential equation. The time constants of the various regulating and stabilizing equipments are assumed and then the gains of the amplifiers in the main and stabilizing loops are determined from stability considerations. The transient performance specifications cannot be incorporated directly into this design. Once the gains and time constants are selected, the system is analysed for design specifications such as settling time, steady state error, maximum overshoot etc. The design is a trial and error process because the regulator parameters are adjusted till the design specifications are nearly met with.

The methods used for the analysis and design of classical control systems are the Routh-Hurwitz and Nyquist criteria and root locus technique. These methods are frequency domain analysis oriented. The specifications must be given in terms of frequency domain characteristics. More recently, the method of sensitivity analysis and Mitrovic method are also employed to obtain better regulator parameters. The classical control techniques can be easily applied only for small systems and that too for linear and time invariant systems. These methods cannot be easily extended to multivariable control systems. One cannot obtain the best possible design by these methods, since controller configurations and the feedback signals are fixed apriori.

1.2 MODERN TRENDS

With the growth of system size, the analysis and design of control systems become more difficult without the use of digital computers. A new approach to the synchronous machine controller design using optimal control theoretic concepts is increasingly used. The synchronous machine control is identified as an optimal control problem. An integrated form of control law is obtained for both frequency and voltage controls. Since the system behaviour is governed by all the system states, it is necessary to take into account the changes in all the variables and thus derive control signals which are functions of all these quantities. The control of voltage and frequency is treated as a single control problem.

In comparison with more conventional methods for feedback control system design, the modern optimal control theory design procedure is more direct because of the inclusion of all the important aspects of performance in a single design index. Also the method can be used to nonlinear as well as time varying systems, however, at the expense of increased computational complexity. The conversion of prescribed design specifications into meaningful mathematical performance index may not be straight forward. But a number of performance indices are suggested in the literature and the nonuniqueness

of them makes the selection of a performance index simpler. The modern control methods require complex computer programmes and a good deal of computing time for nonlinear and time varying systems. This is not a disadvantage in itself because the classical control methods cannot be used as such for the nonlinear and time varying systems.

An optimal regulator obtained in reference 18 for the single machine system uses a simple model for the machine and flux linkages as some of the state variables. The configurations of the governor and voltage regulator are fixed apriori and the inputs to these regulating equipments are obtained by optimal control theory. Also it does not investigate the suitability of the optimal control for large disturbances and it is assumed that all the state variables can be measured for feedback. The reference 20 considers a current model for the machine and the optimal regulator performance is investigated for step load changes. The effect of incomplete state feedback was analysed for impulse disturbances in reference 29.

The progress in power semiconductors has made static excitation of synchronous machine possible using controlled bridge rectifiers. By fast electronic control all time lags except the main generator field time constant are practically eliminated. Electrohydraulic

governors combined with electronic controllers provide better frequency control and improved dynamic performance with lesser governor dead bands compared to conventional mechanical or hydraulic regulators. Therefore the dynamics of the voltage regulator and speed governor need not be considered along with the machine model. This thesis makes a detailed study of the design of optimal and suboptimal controllers and investigates the performance both for small and large perturbations. The physical realizability of the controllers is also discussed.

1.3 SCOPE OF THE THESIS

The synchronous machine control is identified as an optimal control problem. For successfully applying the techniques of optimal control theory, a state space model is derived in a suitable form for the synchronous machine system. The resulting optimal control policies for nonlinear systems are difficult to implement. Hence a linearized state space model is obtained for which a constant feedback optimal control law is derived[21]. A fairly common type of voltage regulator and a speed governor are chosen and the performance of the system is investigated with these regulators. It is shown that the performance is affected largely by the regulator parameters. An improper choice of them may lead to system instability. The conventional methods of

selecting such parameters are discussed. The second method of Lyapunov is employed to determine the optimum values of the regulator gains[16], using a state space model for the machine and regulators.

From investigation of the system performance provided with optimal regulators for large disturbances, it is concluded that the optimal linear feedback control law can be used for the nonlinear system. The implementation of such a control law requires the direct measurement of complete state vector. It is seldom that all of them can be monitored directly and easily. To overcome this difficulty, a compatible dynamic observer is designed which will reconstruct the state vector from the measurable outputs[23]. However, it is found that the cascaded optimal control system with the observer can be used only for small perturbations because the overall feedback control law is different for different operating conditions. Therefore, the question of using a control law which is a linear feedback of the output variables is discussed[28]. The suboptimal control law thus obtained can be used even for large disturbances and hence it can be easily implemented on the nonlinear model.

The normal synchronous machines have limited reactive power capacity and the stability of the system under lightly loaded conditions is poor. The reactive

capacity cannot be improved even by fast acting voltage regulators. By the use of an additional field winding with proper control schemes, the reactive limit can be increased as well as the dynamic performance of the system. A divided winding rotor synchronous machine is discussed which will improve the system performance. An optimal state regulator is obtained for the linearized model and as such it can be used on the nonlinear system[34]. The design of an observer and incomplete state feedback of the d.w.r. machine are investigated. It is found that the suboptimal control law gives a satisfactory performance even for large disturbances and therefore it can be practically implemented.

The stochastic optimal control is an active field of research in the recent times. The synchronous machine control is investigated in the presence of random disturbances in the state and uncertainty in the output measurements. The effect of noise in the measurement of output variables on the overall performance of the system with optimal control is also studied.

For multivariable control system design and analysis by the modern optimization techniques, excessive computer memory and time are required. A simplified model is obtained using Schwarz canonical form for the analysis of large, linear and time invariant systems[41]. The

feasibility of using reduced models to obtain suboptimal controllers for the complete system is also investigated.

1.4 AIM OF THE THESIS

The optimal control theoretic concepts are increasingly applied to the power system design and control in the recent years. The need for the study of synchronous machine voltage and frequency control by optimal regulators becomes important in the light of system complexity both in design and operation. The aim of the thesis is to obtain a mathematical model for the single machine system in a suitable form and then obtain an optimal regulator which can be easily implemented on practical systems. Large interconnected power systems demand complex computation and therefore the question of using simplified models for analysis becomes relevant. The optimal control law obtained for the reduced system will be suboptimal for the original system. Even if the control law is suboptimal the implementation of the same on practical systems should be feasible. These aspects are discussed in the coming chapters.

CHAPTER II

STATE SPACE MODEL OF SYNCHRONOUS MACHINE

2.1 INTRODUCTION

In this chapter, the state space model of a single machine system is derived in a form most suitable for the design of optimal regulators. The study of dynamic behaviour of synchronous machines has followed traditionally the analysis of the piecewise linear model using the characteristic equation of the system by employing classical control techniques. A simple alternative in keeping with the modern system analysis is to formulate the system behaviour, not in terms of a single high order differential equation but in terms of sets of first order equations. Such a mathematical model is most suited for the application of modern control theoretic concepts and system optimization.

A conventional voltage regulator and speed governor configuration is chosen and the state model of the controlled system is then obtained. The performance of the free and controlled system is obtained for an impulse type disturbance on the system. The responses of the free and conventionally controlled systems are compared.

2.2 GENERAL NONLINEAR SYSTEM EQUATIONS

The single machine system shown in Figure 2.1 consists of a salient pole synchronous generator connected to an infinite bus through a long transmission line. The state space model of this system is desired. The inductances of the various machine windings are functions of the instantaneous angular position of the rotor and hence the differential equations describing the machine performance are time varying equations. Park's[2] transformation is applied to the stator quantities to obtain time invariant differential equations. The assumptions made in the development of the system equations are :

1. Saturation and hysteresis in every magnetic circuit and eddy currents in the armature iron are neglected.
2. The stator windings are sinusoidally distributed around the air gap so far as mutual effects between them are concerned.
3. Electrical transients on the transmission system are neglected and also the line parameters are assumed to be invariant with instantaneous system frequency.
4. Mutual inductances between armature and rotor circuits on the same axes are made equal by the proper choice of rotor base quantities.
5. Only two damper bar windings are considered, one in the direct axis and the other in the quadrature axis.

6. Only those modes of operation which do not require zero axis variables are considered.

The geometrical configuration of different windings is shown in Figure 2.2. The performance equations are given below for three phase balanced operation[3,4,5,6]. All the quantities are in p.u. except time and angle which are in seconds and radians respectively. The per unit angular frequency is chosen as unity and hence reactances are used in the flux linkage equations, instead of inductances.

Direct Axis Flux Linkages:

$$\psi_{fd} = x_{ffd}\, i_{fd} - x_{ad}\, i_d + x_{ad}\, i_{kd} \tag{2.1}$$

$$\psi_d = x_{ad}\, i_{fd} - x_d\, i_d + x_{ad}\, i_{kd} \tag{2.2}$$

$$\psi_{kd} = x_{ad}\, i_{fd} - x_{ad}\, i_d + x_{kkd}\, i_{kd} \tag{2.3}$$

Quadrature Axis Flux Linkages:

$$\psi_q = - x_q\, i_q + x_{aq}\, i_{kq} \tag{2.4}$$

$$\psi_{kq} = - x_{aq}\, i_q + x_{kkq}\, i_{kq} \tag{2.5}$$

Direct Axis Voltages:

$$v_{fd} = \frac{1}{w_o}\, p\, \psi_{fd} + r_{fd}\, i_{fd} \tag{2.6}$$

$$v_d = \frac{1}{w_o}\, p\, \psi_d - r_a\, i_d - \frac{w}{w_o}\, \psi_q \tag{2.7}$$

$$0 = \frac{1}{w_o}\, p\, \psi_{kd} + r_{kd}\, i_{kd} \tag{2.8}$$

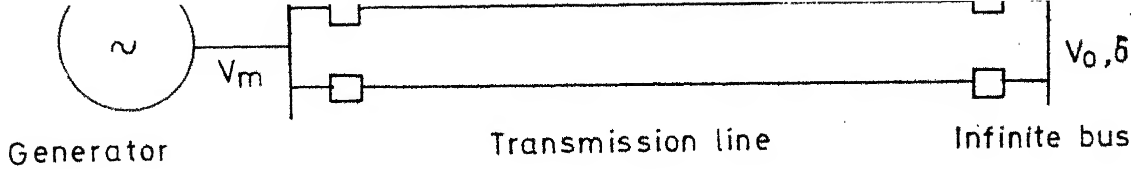

FIG. 2·1 SINGLE MACHINE SYSTEM

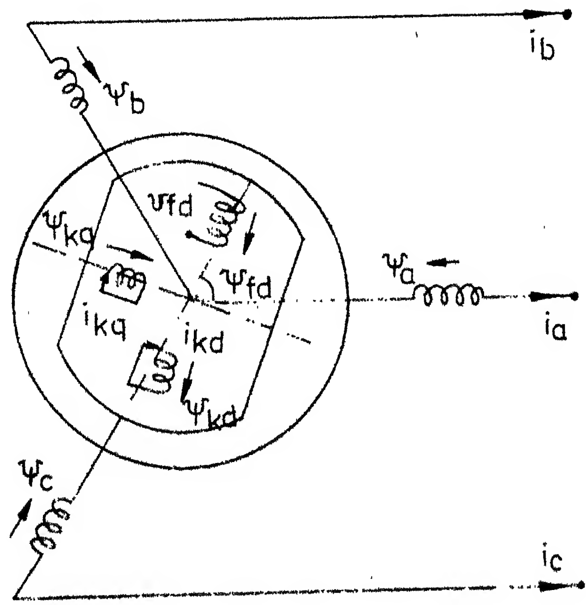

FIG. 2·2 SCHEMATIC DIAGRAM OF SYN. GENERATOR

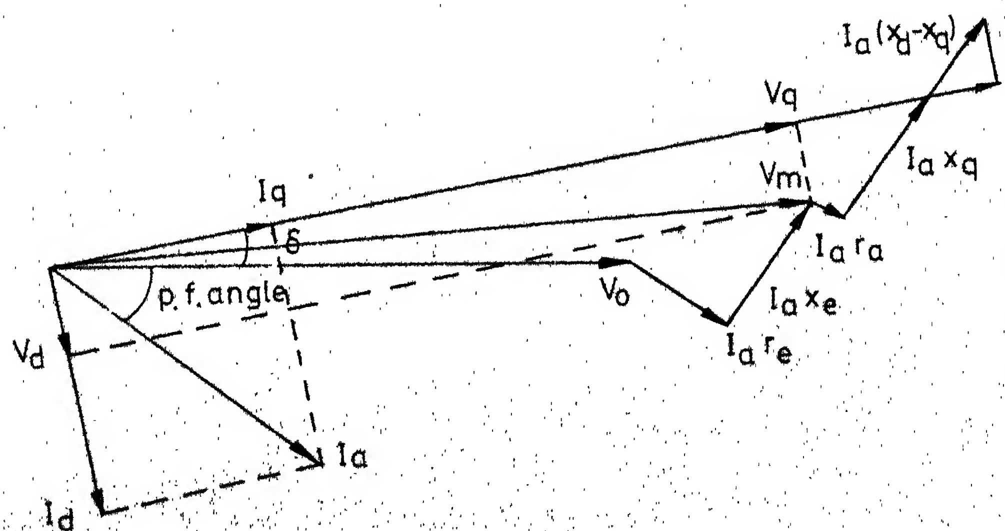

FIG. 2·3 PHASOR DIAGRAM OF SYN. GENERATOR

Quadrature Axis Voltages:

$$v_q = \frac{1}{w_o} p \psi_q - r_a i_q + \frac{w}{w_o} \psi_d \qquad (2.9)$$

$$0 = \frac{1}{w_o} p \psi_{kq} + r_{kq} i_{kq} \qquad (2.10)$$

Electrical Torque in the Air Gap:

$$T_e = \psi_d i_q - \psi_q i_d \qquad (2.11)$$

Generator Terminal Voltage:

$$V_m^2 = v_d^2 + v_q^2 \qquad (2.12)$$

The transmission line equations are obtained referring to the phasor diagram (Figure 2.3) as

$$v_d = V_o \operatorname{Sin} \delta + r_e i_d - x_e i_q \qquad (2.13)$$

$$v_q = V_o \operatorname{Cos} \delta + x_e i_d + r_e i_q \qquad (2.14)$$

Dynamic Equation of Motion of Rotor:

$$M \frac{d^2 \delta}{dt^2} = T_i - T_e - K_d \frac{d \delta}{dt} \qquad (2.15)$$

The variables used in the above equations are explained in the nomenclature.

The set of equations (2.1) to (2.15) are manipulated to obtain the state space model. The state variables are chosen as the winding currents in the d and q axis of the stator and rotor, the rotor angular velocity and rotor angle. The winding currents are chosen as some of the

state variables instead of winding flux linkages because this seems to be a natural choice when machines are to be interconnected through a transmission network where the performance may be more readily visualized in terms of voltages and currents rather than in terms of flux linkages. The implementation of optimal regulators requires the measurement of state vector for feedback. Thus the current variables offer definite advantage over flux linkage variables as the former variables are comparatively easy to measure than the later ones. The machine terminal voltage, rotor angle and rotor angular velocity are assumed to be the measurable output variables. The control variables are selected as the input torque T_1 and the field voltage E_{fd}, referred to the stator side. Substituting currents for the flux linkages from equations (2.1) to (2.5), the following equations are obtained :

$$p\,\delta = w - w_o \tag{2.16}$$

$$pw = [T_1 - K_d(w - w_o) - x_{ad}\, i_{fd}\, i_q - x_{ad}\, i_{kd}\, i_q + (x_d - x_q) i_d\, i_q + x_{aq}\, i_{kq}\, i_d]/M \tag{2.17}$$

$$x_{ffd}\, p\, i_{fd} - x_{ad}\, p\, i_d + x_{ad}\, p\, i_{kd} = w_o \frac{r_{fd}}{x_{ad}} E_{fd} - w_o\, r_{fd}\, i_{fd} \qquad \ldots (2.18)$$

$$x_{ad}\, p\, i_{fd} - x_d\, p\, i_d + x_{ad}\, p\, i_{kd} = w_o\, r_a\, i_d + w_o\, V_o \sin\delta + w_o\, r_e\, i_d - w_o\, x_e\, i_q + w(x_{aq}\, i_{kq} - x_q\, i_q) \qquad \ldots (2.19)$$

$$x_{ad}\, p\, i_{fd} - x_{ad}\, p\, i_d + x_{kkd}\, p\, i_{kd} = - w_o\, r_{kd}\, i_{kd} \qquad (2.20)$$

$$- x_q\, p\, i_q + x_{aq}\, p\, i_{kq} = w_o (V_o \operatorname{Cos} \delta + x_e\, i_d + r_e\, i_q + r_a\, i_q) - w(x_{ad}\, i_{fd} - x_d\, i_d + x_{ad}\, i_{kd}) \quad \ldots \qquad (2.21)$$

$$- x_{aq}\, p\, i_q + x_{kkq}\, p\, i_{kq} = - w_o\, r_{kq}\, i_{kq} \qquad (2.22)$$

$$\delta = \delta \qquad (2.23)$$

$$w = w \qquad (2.24)$$

$$V_m^2 = (V_o \operatorname{Sin} \delta + r_e\, i_d - x_e\, i_q)^2 + (V_o \operatorname{Cos} \delta + x_e\, i_d + r_c\, i_q)^2 \qquad (2.25)$$

The nonlinear state model of the system can be obtained from equations (2.16) to (2.25) as

$$E\, \dot{X} = f(X) + D\, u \qquad (2.26)$$

$$Y = g(X) \qquad (2.27)$$

where the state vector X, the output vector Y and the control vector u are given by

$$X = (\delta, w, i_{fd}, i_d, i_{kd}, i_q, i_{kq})^T$$

$$Y = (\delta, w, V_m)^T$$

$$u = (E_{fd}, T_i)^T$$

The matrices E and D are given by

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_{ffd} & -x_{ad} & x_{ad} & 0 & 0 \\ 0 & 0 & x_{ad} & -x_d & x_{ad} & 0 & 0 \\ 0 & 0 & x_{ad} & -x_{ad} & x_{kkd} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -x_q & x_{aq} \\ 0 & 0 & 0 & 0 & 0 & -x_{aq} & x_{kkq} \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & w_o\, r_{fd}/x_{ad} & 0 & 0 & 0 & 0 \\ 0 & 1/M & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

The vectors f(X) and g(X) are given by

$$f(X) = \begin{bmatrix} w - w_o \\ [- K_d(w - w_o) - x_{ad}\, i_{fd}\, i_q - x_{ad}\, i_{kd}\, i_q + \\ \quad x_{aq}\, i_{kq}\, i_d + (x_d - x_q)\, i_d\, i_q]/M \\ - w_o\, r_{fd}\, i_{fd} \\ w_o\, V_o\, \mathrm{Sin}\,\delta + w_o(r_a + r_e)\, i_d - (w\, x_q + w_o\, x_e) i_q \\ \quad + w\, x_{aq}\, i_{kq} \\ - w_o\, r_{kd}\, i_{kd} \\ w_o\, V_o\, \mathrm{Cos}\,\delta + (w_o\, x_e - w\, x_d)\, i_d + w_o(r_a + r_e) i_q \\ \quad - w\, x_{ad}\, i_{fd} - w\, x_{ad}\, i_{kd} \\ - w_o\, r_{kq}\, i_{kq} \end{bmatrix}$$

$$g(X) = \begin{bmatrix} \delta \\ w \\ \left[(V_o \sin\delta + r_e i_d - x_e i_q)^2 + (V_o \cos\delta + x_e i_d + r_e i_q)^2 \right]^{\frac{1}{2}} \end{bmatrix}$$

2.3 LINEARIZED STATE SPACE MODEL

Except for a very few special cases, there are no exact analytical methods for analysing nonlinear systems. Practical ways of solving nonlinear problems involve either graphical or experimental approaches. Another method of analysis is the piecewise linear approach. The analysis of linearized model has been extensively studied in the literature. Further the results obtained from the study of linear model can be extended with some approximations to the nonlinear case. It is desired to obtain a linear feedback control law which is easy to implement either on the linear model or nonlinear model. To apply the techniques of optimal control theory it is convenient to represent the system as a linear time invariant system. Hence in this thesis, the study of linear system is considered.

Using Taylor series expansion of equations (2.26) and (2.27) about an operating point and retaining the first order terms in the expansion, the state space equations reduce to

$$E\dot{X} = F X + D u \tag{2.28}$$

$$Y = C X \tag{2.29}$$

where X, Y and u are the deviations of the state, output and control vectors respectively about the given operating point considered. The matrices F and C are given by

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -K_d/M & -x_{ad}i_{qo}/M & f_{24} & -x_{ad}i_{qo}/M & f_{26} & -x_{aq}i_{do}/M \\ 0 & 0 & -w_o r_{fd} & 0 & 0 & 0 & 0 \\ w_o V_o \mathrm{Cos}\,\delta_o & f_{42} & 0 & w_o(r_a+r_e) & 0 & -w_o(x_q+x_e) & w_o x_{aq} \\ 0 & 0 & 0 & 0 & -w_o r_{kd} & 0 & 0 \\ -w_o V_o \mathrm{Sin}\,\delta_o & f_{62} & -w_o x_{ad} & w_o(x_d+x_e) & -w_o x_{ad} & w_o(r_a+r_e) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -w_o r_{kq} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ c_{31} & 0 & 0 & c_{34} & 0 & c_{36} & 0 \end{bmatrix}$$

where

$$f_{24} = -[(x_d - x_q)\,i_{qo} + x_{aq}\,i_{kqo}]\,/M$$

$$f_{26} = -[x_{ad}\,i_{fdo} + x_{ad}\,i_{kdo} + (x_d - x_q)i_{do}]\,/M$$

$$f_{42} = x_{aq}\,i_{kqo} - x_q\,i_{qo}$$

$$f_{62} = x_d\,i_{do} - x_{ad}(i_{fdo} + i_{kdo})$$

$$c_{31} = V_o(v_{do}\,\mathrm{Cos}\,\delta_o - v_{qo}\,\mathrm{Sin}\,\delta_o)/V_{mo}$$

$$c_{34} = (v_{do}\,r_e + v_{qo}\,x_e)/V_{mo}$$

$$c_{36} = (v_{qo}\,r_e - v_{do}\,x_e)/V_{mo}$$

The equation (2.28) is premultiplied by the inverse of E to obtain the system state model as

$$\dot{X} = A X + B u \tag{2.30}$$

where

$$A = E^{-1} F \quad \text{and} \quad B = E^{-1} D$$

Thus the state space model in the required form for the single machine infinite bus system is given by equations (2.29) and (2.30) which are used in the coming chapters.

The system chosen has the following parameters where all the quantities are given in p.u. except time and angle which are in seconds and radians respectively.

Synchronous Machine Parameters[4]:

$x_{ad} = 1.0$ $x_{aq} = 0.6$ $x_d = 1.2$ $x_q = 0.8$

$x_{ffd} = 1.1$ $x_{kkd} = 1.1$ $x_{kkq} = 0.8$ $x_{al} = 0.2$

$r_a = 0.01$ $r_{kd} = 0.02$ $r_{kq} = 0.04$ $r_{fd} = 0.0011$

$w_o = 314.0$ $M = 0.0192$ $K_d = 0.0032$ $V_o = 1.0$

Transmission Line Data:

$r_e = 0.05$ $x_e = 0.3$ for both lines

The operating point data, when the machine is delivering rated KVA at 0.8 p.f. lagging to infinite bus, referring to phasor diagram of Figure 2.3, are obtained as

$\delta_o = 26.3°$ $i_{do} = 0.8921$ $i_{qo} = 0.452$ $i_{fdo} = 2.2614$

$v_{do} = 0.3526$ $v_{qo} = 1.1864$ $V_{mo} = 1.2377$ $i_{kdo} = i_{kqo} = 0$

For the above operating conditions, the matrices A, B and C are calculated and are given by

$$A = \begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.1667 & -23.54 & 9.42 & -23.54 & -99.20 & 27.88 \\ -541.18 & 0.6953 & -2.13 & -36.23 & 24.15 & 664.23 & -362.31 \\ -1136.48 & 1.4601 & -0.66 & -76.08 & -12.08 & 1394.88 & -760.85 \\ -541.18 & 0.6953 & 1.33 & -36.23 & -38.65 & 664.23 & -362.31 \\ 397.98 & 3.4027 & 897.14 & -1345.71 & 897.14 & -53.83 & -26.91 \\ 298.48 & 2.5520 & 672.86 & -1009.29 & 672.86 & -40.37 & -35.88 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.00 & 0.00 & 2.1255 & 0.6642 & -1.3285 & 0.00 & 0.00 \\ 0.00 & 52.08 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -0.17 & 0.00 & 0.00 & 0.32 & 0.00 & -0.04 & 0.00 \end{bmatrix}$$

By checking the real part of the eigen values of the system matrix A, it is found that the free system is stable. The response of the free system obtained by digital simulation, for an initial perturbation in the field current of 0.05 p.u. is shown in Figure 2.4. The response of the system is rather poor because it is too oscillatory with more overshoot and takes more than 10 seconds to settle down. The response can be improved by having proper controls.

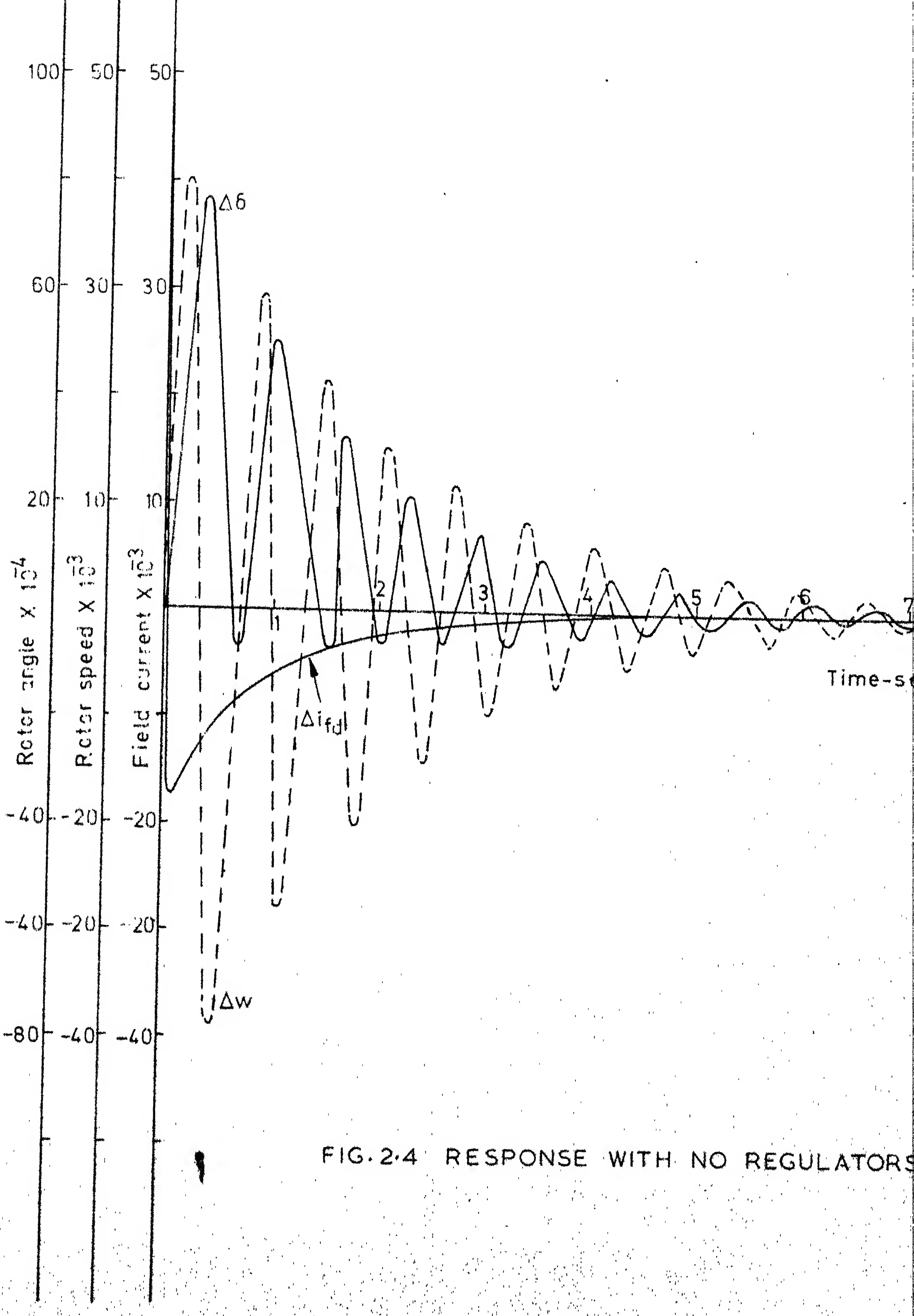

FIG. 2.4 RESPONSE WITH NO REGULATORS

2.4 PERFORMANCE WITH CONVENTIONAL REGULATORS

In the conventional design procedure of regulators, suitable configurations for the regulating and stabilizing equipments are selected apriori. Then the system is analyzed assuming various time constants, to determine the overall loop gain from stability considerations. This gain is distributed to various components in the control loop depending upon their physical feasibility. The gains and time constants are further adjusted until the system specifications such as overshoot, settling time etc. are satisfied.

A fairly common type of voltage regulator and speed governor are shown in Figures 2.5 and 2.6 respectively. The voltage regulator has a stabilizer with gain K_s and time constant T_s, for the amplifier in the main loop. The speed governor has two time constants T_g and T_h representing the governor and turbine time constants. The performance equations for the regulators are given by[5]

Voltage Regulator:

$$p\, E_{fd} = \frac{K_v}{T_v}(V_{ref} - V_m - V_s) - \frac{1}{T_v} E_{fd} \qquad (2.31)$$

$$p\, V_s = \frac{K_s}{T_s}\, p\, E_{fd} - \frac{1}{T_s} V_s \qquad (2.32)$$

Speed Governor:

$$p^2\, T_i = \frac{K_g}{w_o T_g T_h}(w_o - w) - \frac{(T_g + T_h)}{T_g T_h}\, p\, T_i - \frac{1}{T_g T_h} T_i \qquad \ldots (2.33)$$

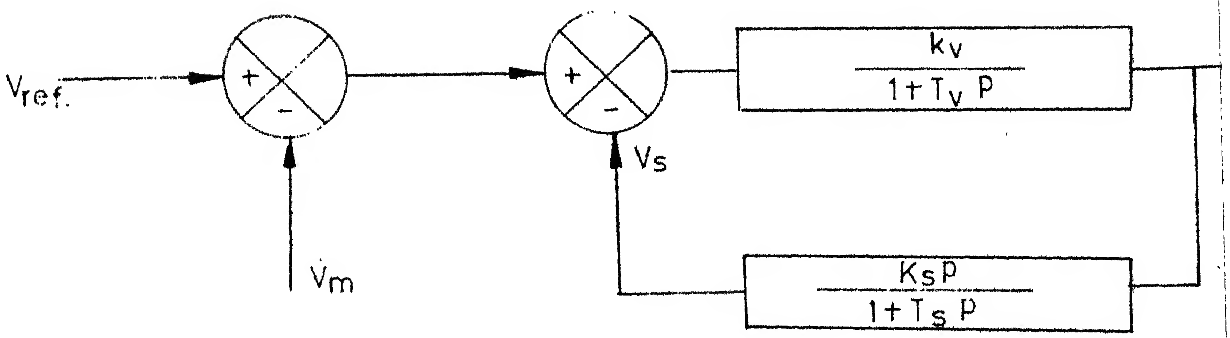

FIG. 2·5 CONVENTIONAL VOLTAGE REGULATOR

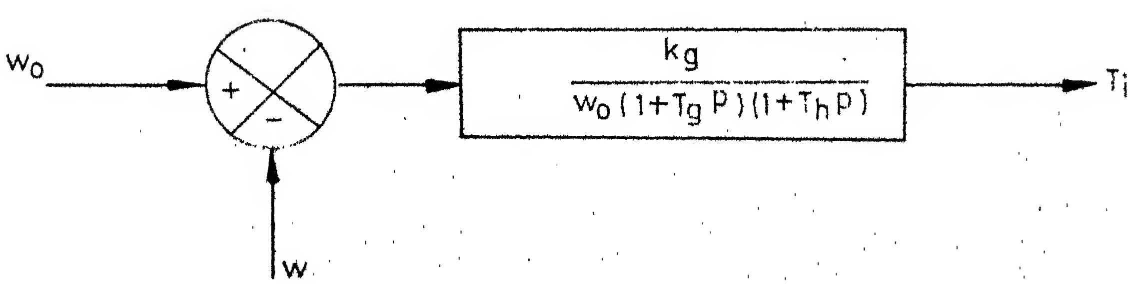

FIG. 2·6 CONVENTIONAL SPEED GOVERNOR

The above equations (2.31) to (2.33) describing the operation of controlling equipments can be linearized and added to the system equation (2.30). Choosing the state variables X_8, X_9, X_{10} and X_{11} as ΔE_{fd}, ΔV_s, ΔT_1 and $p\Delta T_1$ respectively, the state equations for the regulators are given by

$$\dot{X}_8 = a_1(c_{31}\, X_1 + c_{34}\, X_4 + c_{36}\, X_6 + X_9) - \frac{1}{T_v} X_8 \qquad (2.34)$$

$$\dot{X}_9 = a_2(c_{31}\, X_1 + c_{34}\, X_4 + c_{36}\, X_6 + \frac{1}{K_v} X_8) + (a_2 - \frac{1}{T_s})X_9 \qquad (2.35)$$

$$\dot{X}_{10} = X_{11} \qquad (2.36)$$

$$\dot{X}_{11} = a_3 \cdot \frac{K_g}{W_o} X_2 + a_3(T_g + T_h)X_{11} + a_3\, X_{10} \qquad (2.37)$$

where

$$a_1 = -\frac{K_v}{T_v} \qquad a_2 = a_1 \frac{K_s}{T_s} \qquad a_3 = -\frac{1}{T_g\, T_h}$$

Combining controller equations (2.34) to (2.37) with system equation (2.30) the state space model for the conventionally controlled system becomes

$$\dot{X} = A_1\, X \qquad (2.38)$$

The following regulator parameters are chosen[5] as

$K_v = 5.0$ $\quad K_s = 0.04$ $\quad K_g = 20.0$

$T_v = 1.0$ $\quad T_s = 0.5$ $\quad T_g = 0.5$ $\quad T_h = 0.5$

The system matrix A_1 for the operating point considered in the last section with the above parameters is given by

$$A_1 = \begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.167 & -23.54 & 9.42 & -23.54 & -99.00 & 27.88 & 0.00 & 0.00 & 52.08 & 0.00 \\ -541.18 & 0.6953 & -2.13 & -36.23 & 24.15 & 664.23 & -362.31 & 2.125 & 0.00 & 0.00 & 0.00 \\ -1136.48 & 1.4601 & -0.66 & -76.08 & -12.08 & 1394.88 & -760.85 & 0.664 & 0.00 & 0.00 & 0.00 \\ -541.18 & 0.6953 & 1.33 & -36.23 & -38.65 & 664.23 & -362.31 & -1.328 & 0.00 & 0.00 & 0.00 \\ 397.98 & 3.4027 & 897.14 & -1345.71 & 897.14 & -53.83 & -26.91 & 0.00 & 0.00 & 0.00 & 0.00 \\ 298.48 & 2.5520 & 672.86 & -1009.29 & 672.86 & -40.37 & -35.88 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.849 & 0.00 & 0.00 & -1.51 & 0.00 & 0.187 & 0.00 & -1.00 & -5.00 & 0.00 & 0.00 \\ 0.068 & 0.00 & 0.00 & -0.121 & 0.00 & 0.015 & 0.00 & -0.08 & -2.40 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & -0.2548 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -4.00 & -4.00 \end{bmatrix}$$

The controlled system is again solved for the performance when there is an initial disturbance in the field current of 0.05 p.u. The response obtained by digital simulation using Runge-Kutta fourth order integration method is shown in Figure 2.7. The response is still oscillatory and takes about 8 seconds to settle down. An improper choice of the regulator parameters may sometimes render the system unstable.

2.5 CONCLUSION

An accurate state space model of the synchronous machine system is derived which is most suitable for the application of optimal control methods. The systematic method of obtaining the state space model is indicated. The configurations for the voltage regulator and speed governor are selected and the performance of the regulated system is compared with unregulated system. Since the regulator parameters have marked effect on the system performance, proper parameter values are necessary. Methods of selecting these parameters are discussed in the next chapter.

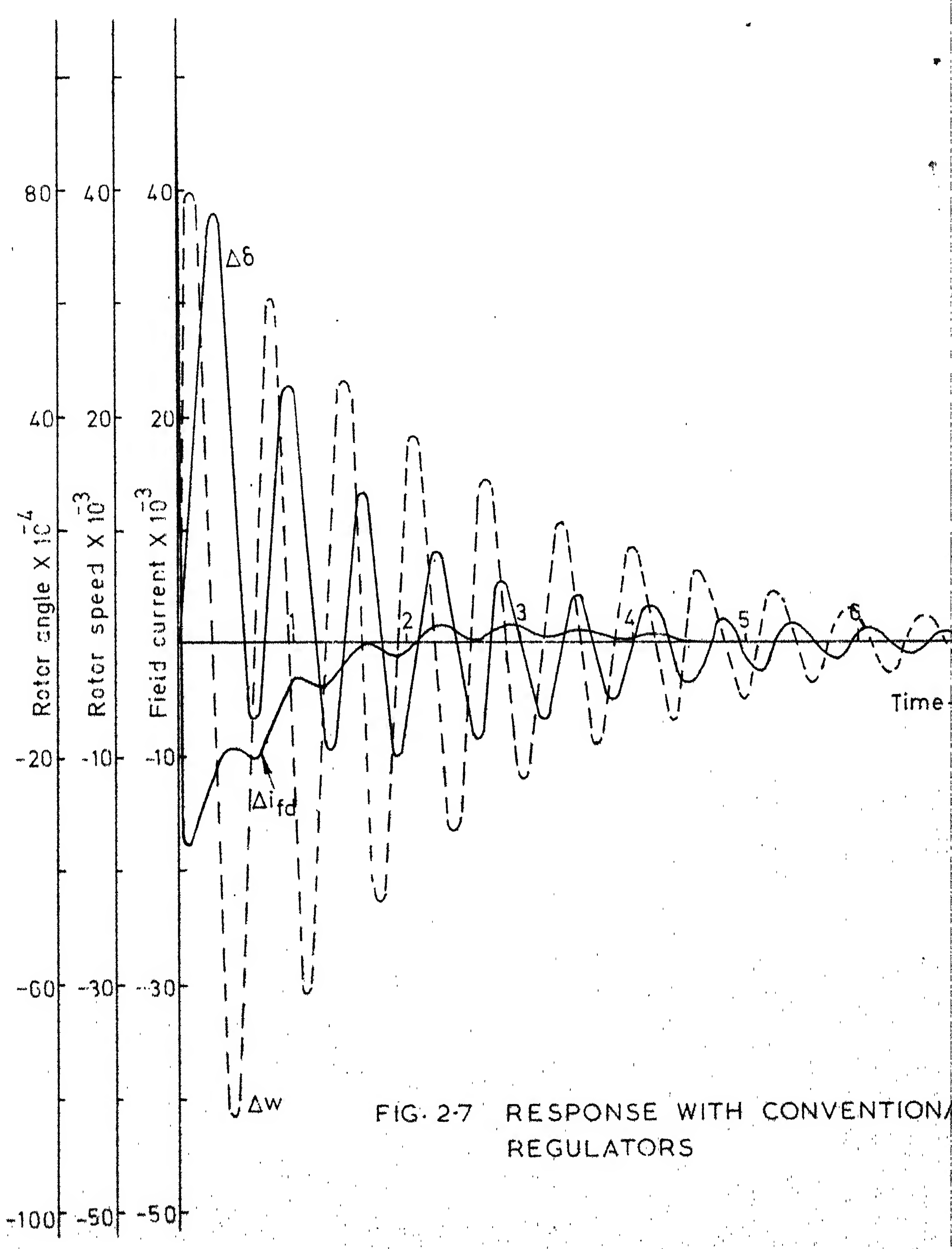

FIG. 2·7 RESPONSE WITH CONVENTIONA[L]
REGULATORS

CHAPTER III

OPTIMAL REGULATOR GAINS BY SECOND METHOD OF LYAPUNOV

3.1 INTRODUCTION

The optimum selection of voltage regulator and speed governor gains, is discussed by using Lyapunov's second method so that the settling time is minimum in the event of system disturbance. The voltage regulator and speed governor configurations are chosen apriori. Different methods of analysis and design of synchronous machine regulators using conventional control techniques are discussed. The performance of the system with optimal regulator parameters is then obtained for comparison with the response given in Chapter II.

3.2 METHODS OF DESIGN AND ANALYSIS OF CONVENTIONAL REGULATORS

With increasing system voltages and generator unit size for economical reasons, the loss of stability in the dynamic region of operation of any one machine becomes a more serious problem. Properly designed excitation systems with voltage regulators of the continuously acting type can raise the stability limits considerably. Greater reliance is therefore placed on well designed control systems which extend the operating margin of stability.

Once the mathematical model of the system including the alternator and regulators are established, the classical control methods can be applied for the analysis of the behaviour of the system with regulators. Since the development of 2-axis theory by R.H. Park in 1929, the general effect of regulators on the stability limit under particular operating conditions have been discussed by many authors. The methods used for the selection of regulator parameters which give a better performance are Routh-Hurwitz and Nyquist criterion, root locus techniques, sensitivity and Mitrovic methods.

Concordia[7] used the generalized machine theory for modelling the system and the well known Routh's criterion to determine the stability limit of round rotor machines under unity power factor operating conditions.

Messerly[8] extended the analysis to include both voltage regulator and speed governor, which means control of both terminal voltage and rotor speed and used the transfer function method of analysis.

Root locus method[9] was used to study the effect of regulator parameters on the stability by plotting the loci of the roots of the system characteristic equation. A qualitative assesment of the system transient performance can be made by the study of the root locus diagrams.

Using the system sensitivity equations[10], the sensitivity of each of the characteristic roots to different parameter values can be obtained and a region in the parameter plane can be determined within which the system is stable.

Using generalized Mitrovic method[11] formulas are obtained for the calculation of optimum loop gains and damping gains of a certain class of voltage regulators for better steady state stability. The generalized Mitrovic method is also employed[12] to obtain a prescribed root configuration in the s-plane by variation of the regulator parameters.

In most of these methods the system model is obtained as a single higher order differential equation, including the regulator dynamics. Using classical control techniques the system characteristic equation is tested for stability. The parameters are varied till a satisfactory performance is obtained. In this chapter, the second method of Lyapunov which relies heavily on the state variable formulation, is used to obtain optimum regulator parameters. The design criterion is that the settling time of the system variables should be minimum in the event of system disturbances.

3.3 LYAPUNOV FUNCTION AND MINIMUM SETTLING TIME

The Lyapunov's second method can be used to design optimal regulators which ensure an asymptotically stable

system. Also the system output will be continuously driven towards the desired value. The Lyapunov's method requires the utilization of a continuous scalar function of state variables called the Lyapunov function $V(X)$, in conjunction with the system state equation. Depending upon the properties of V-function and its time derivative $\dot{V}(X)$, the stability or instability of the equilibrium state can be proved.

A Lyapunov function gives a measure of the system state at any given instant of time and hence it can be considered as a measure of the distance of the state of the system from the equilibrium state, in the state space. If a V-function is known for an autonomous system, then it can be used to estimate the rapidity of the transient response or the rate at which the state comes back to its normal state from the disturbed state. A measure of the transient response can be taken as the normalized rate at which the V-function changes.

Considering the origin as a stable equilibrium state, let[15]

$$\eta \leq \underset{X}{\mathrm{Min}} \; \frac{-\dot{V}(X)}{V(X)} \tag{3.1}$$

in some region of the state space excluding the origin. Equation (3.1) is integrated between limits 0 to t assuming η to be constant in the specified region, to give

$$V(X) \leq V(X_o) \exp(-\int_o^t \eta \, dt) \qquad (3.2)$$

The above equation gives a measure of how fast the origin is reached from any given initial state X_o. The time constant[14] for the variation of V-function can be considered as $1/\eta$. Thus a large value of η corresponds to faster response. For a given system, a large number of V-functions can be determined. Hence the largest value of η should be taken as a figure of merit for the system transient performance.

For a linear time invariant system

$$\dot{X} = A_1 X, \; X(o) = X_o \qquad (3.3)$$

a Lyapunov function can be determined easily as a positive definite quadratic form in the state variables, with a positive definite Q matrix, as

$$V(X) = X^T P X \qquad (3.4)$$

where P is the positive definite solution of

$$P A_1 + A_1^T P + Q = 0 \qquad (3.5)$$

and with a negative definite $\dot{V}$-function as

$$\dot{V}(X) = - X^T Q X \qquad (3.6)$$

The figure of merit for the linear system given by equation (3.3) can be defined as[14]

$$\eta = \underset{X}{\mathrm{Min}}(X^T Q X) \qquad (3.7)$$

subject to the constraint

$$X^T P X = 1 \tag{3.8}$$

The minimization of η can be achieved by using Lagrangian multiplier technique, with the Lagrangian multiplier λ. The Hamiltonian function is formed as

$$H(X, \lambda) = X^T Q X + \lambda (1 - X^T P X) \tag{3.9}$$

Minimization of η is the same as minimizing equation (3.9) without any constraint. Thus minimizing H, gives

$$(Q - \lambda P) X = 0 \quad \text{at} \quad X = X_{min} \tag{3.10}$$

Hence

$$X_{min}^T Q X_{min} = \lambda X_{min}^T P X_{min} \tag{3.11}$$

i.e. $\qquad \eta = \lambda > 0 \qquad (3.12)$

From equation (3.10), λ is shown[15] to be the eigen value of $Q P^{-1}$. Thus η can be taken as the minimum eigen value of $Q P^{-1}$. Since a large number of V-functions can be found for the system, η should be taken as the largest value from the set of minimum eigen values of $Q P^{-1}$. The problem is the same as finding the minimum value of the largest eigen values of $P Q^{-1}$. Thus the figure of merit for the linear system is obtained as[16]

$$\eta = \text{Min } [\lambda_{max}(P Q^{-1})] \tag{3.13}$$

3.4 SOLUTION OF THE PROBLEM

The system shown in Figure 2.1 is provided with simple voltage regulator and speed governor. The system model in linear time invariant form as obtained in Chapter II is given by

$$\dot{X} = A X + B u \tag{3.14}$$

The voltage regulator and speed governor dynamics are chosen as

Voltage Regulator:

$$E_{fd} = \frac{K_v}{(1+T_v\ p)} (V_{ref} - V_m) \tag{3.15}$$

Speed Governor:

$$T_i = \frac{K_g}{(1+T_g\ p)} (w_o - w) \tag{3.16}$$

The regulator equations (3.15) and (3.16) are linearised about the same operating point considered in Chapter II and then augmented with the system model given by equation(3.14), taking ΔE_{fd} and ΔT_i as additional state variables. Thus the state space model for the controlled system is obtained in the form of equation (3.3).

The problem facing solution is that the voltage regulator and speed governor gains K_v and K_g respectively, have to be optimized so that the settling time of system variables is minimum in the event of any system disturbance.

The time constants T_v and T_g are selected as $T_v = 2.0$ and $T_g = 0.1$. For the operating point considered in Chapter II with rotor angle of 26.3°, the matrix A_1 is given in the next page.

The Q-matrix is chosen as Q = dia(10,10,10,10,10, 10,10,10,10), and the matrix P is calculated from the equation (3.5). The optimum values of K_v and K_g are obtained by minimizing the largest eigen value of $P Q^{-1}$. Evaluation of the largest eigen value of $P Q^{-1}$ instead of finding the smallest eigen value of $Q P^{-1}$, avoids the computation of P^{-1} in each iteration and hence saves much computing time. The optimum values obtained by Rosenbrock's hill climbing technique[42] (Appendix A) are given by

$$K_v = 3.7584 \qquad K_g = 4.711$$

and the minimum value of $\eta = 170.19465$. The system with these optimal parameters is simulated on the digital computer for the transient response when there is an initial disturbance in the field current of 0.05 p.u. The minimum settling time response is shown in Figure 3.1. The superiority of this response is established by comparison with Figure 2.7. The settling time is very less compared to the previous one.

The voltage regulator configuration with stabilizer shown in Figure 2.5 and governor of the linear type given by

$$T_i = K_g \frac{w - w_o}{w_o} \qquad (3.17)$$

$$
A_1 = \begin{bmatrix}
0.00 & 1.000 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & -0.167 & -23.54 & 9.42 & -23.54 & -59.20 & 27.88 & 0.00 & 52.08 \\
-541.20 & 0.695 & -2.13 & -36.23 & 24.15 & 664.20 & -362.31 & 2.13 & 0.00 \\
-1136.43 & 1.460 & -0.66 & -76.08 & -12.08 & 1394.88 & -760.85 & 0.66 & 0.00 \\
-541.18 & 0.695 & 1.33 & -36.23 & -38.65 & 664.23 & -362.31 & -1.33 & 0.00 \\
397.98 & 3.403 & 897.14 & -1345.71 & 897.14 & -53.83 & -26.91 & 0.00 & 0.00 \\
298.48 & 2.552 & 672.86 & -1009.29 & 672.86 & -40.37 & -35.88 & 0.00 & 0.00 \\
0.085K_v & 0.000 & 0.00 & -0.151K_v & 0.00 & 0.019K_v & 0.00 & -0.50 & 0.00 \\
0.00 & -10.0K_g & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -10.00
\end{bmatrix}
$$

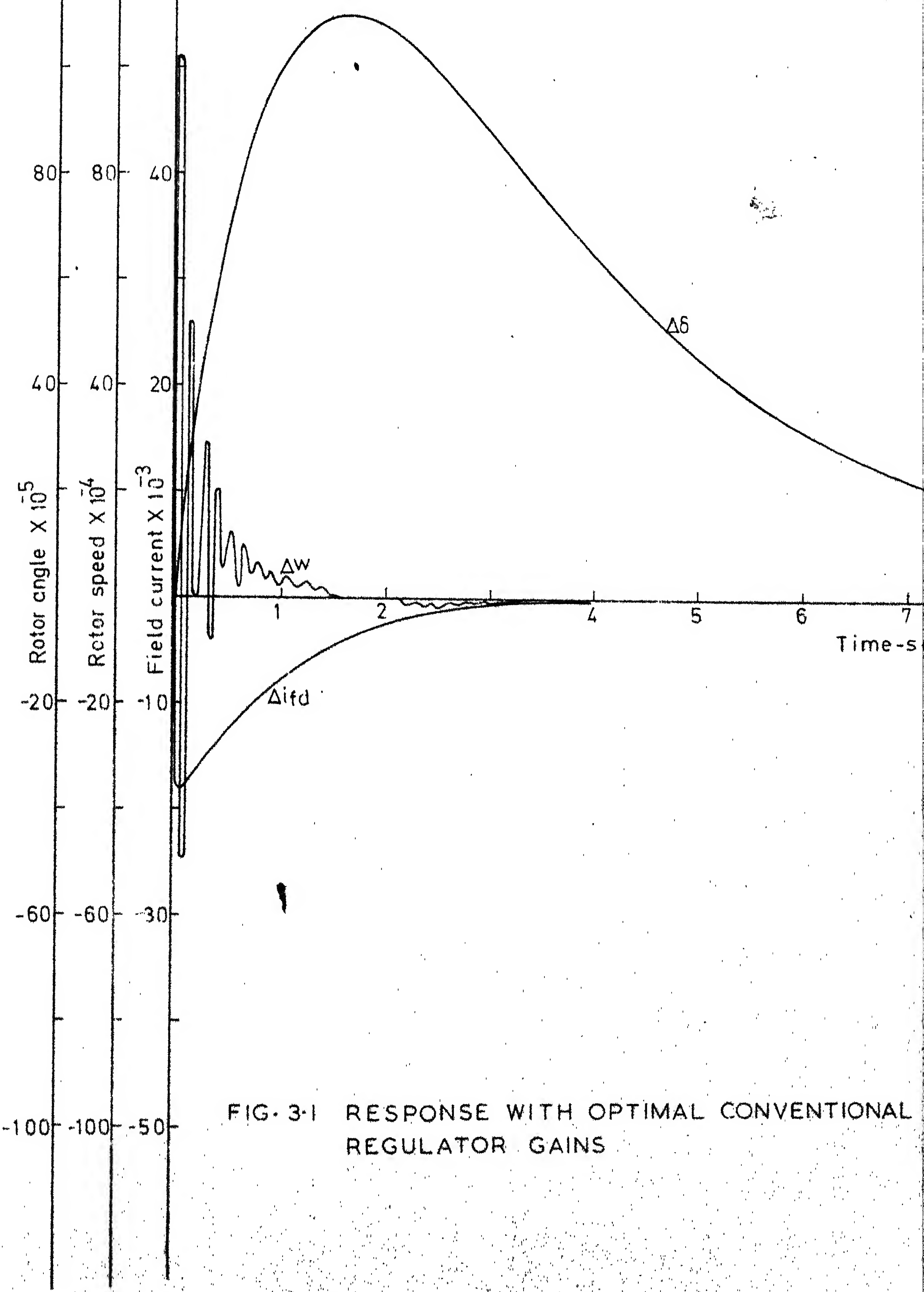

FIG. 3·1 RESPONSE WITH OPTIMAL CONVENTIONAL REGULATOR GAINS

is then considered for selection of optimal parameters. The gains K_g and K_v are assumed to be the variables of interest. The other regulator parameters are taken as given in Chapter II. The optimal gains are obtained as K_v = [illegible].0 and $K_g = 273.0$. With these parameter values the minimum settling time response is obtained as shown in Figure 3.2. Only simple regulator configurations as given by equations (3.15) to (3.17) are considered to illustrate the principle. This method is applicable even when more than two variables (adjustable gains and/or time constants) are involved in the minimization procedure.

3.5 CONCLUSION

The various methods of selecting conventional regulator parameters are briefly discussed. The second method of Lyapunov is used to obtain the optimal regulator gains. The performance criterion is taken as the minimum settling time without much overshoot. An improved performance can be obtained by this design procedure. The stability of the system is ensured automatically by a positive definite solution of the Lyapunov matrix equation. The method can be extended to nonlinear systems easily. But the determination of a suitable Lyapunov function for such high order nonlinear systems is difficult. There are no systematic methods which can be applied easily to obtain V-functions for such nonlinear systems.

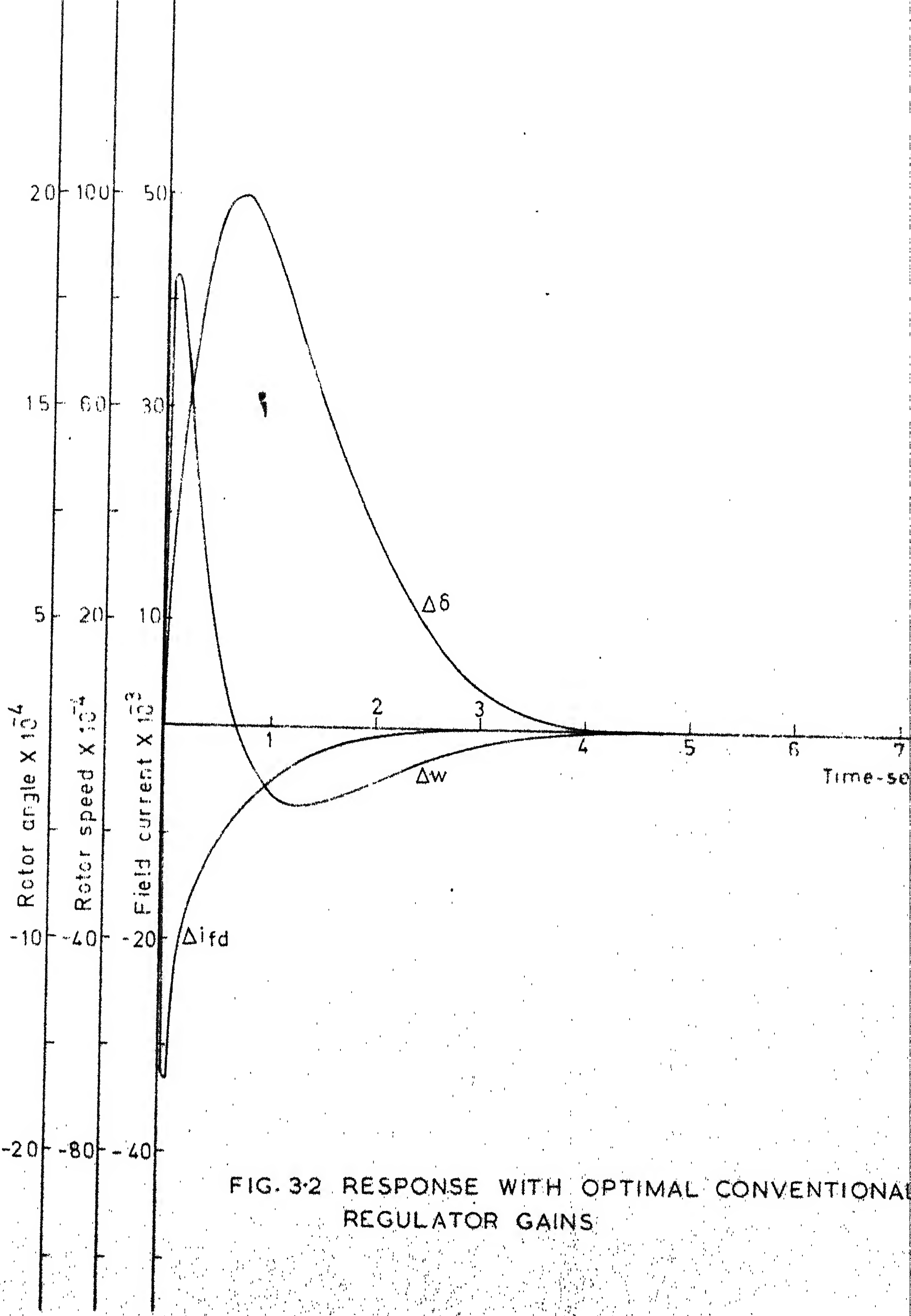

FIG. 3·2 RESPONSE WITH OPTIMAL CONVENTIONAL REGULATOR GAINS

In this chapter the configuration for voltage regulators and speed governors are assumed apriori and independently and then their optimal parameters are obtained. Hither to this has been the practice. However an integrated form of control law which is a combination of all the state variables of the system which provides an optimal response will be very useful because it does not require any apriori knowledge of the regulator configurations. If this control law is linear and time invariant then it will be possible for practical implementations. The following chapters deal with the determination of this type of controllers.

CHAPTER IV

OPTIMAL CONTROL OF SYNCHRONOUS MACHINE

4.1 INTRODUCTION

In this chapter a new approach to the design of excitation and primemover control of the synchronous machine is considered. The optimal output regulator is discussed for both finite and infinite time interval of optimization. The optimal control law is obtained as a linear feedback control of the complete state vector, using the linear time invariant state model derived in Chapter II. The performance of the optimally controlled system is compared with the conventional regulator response for impulse type disturbances. The dynamic performance of the system is also investigated for large disturbances such as line reclosure. The optimal control law is obtained at different operating conditions and performances at different loads are then compared.

4.2 FORMULATION OF OPTIMAL CONTROL PROBLEM

The feedback control of synchronous machines has been given a strong impetus by the modern optimization theory as developed by Pontryagin, Kalman etc. which relies heavily on the state space formulation. It is usually difficult to translate the given system specifications in the conventional design procedure. However,

in the modified approach, an optimal control law is obtained by a suitable choice of a performance criterion. One of the difficulties of controlling a nonlinear system with nonlinear controls is that the optimal control policies are not generally easy to implement.

The system shown in Figure 2.1 is identified as an optimal control problem. The objective is to obtain a linear constant feedback control law which will transfer the system from the given initial state to the desired state. In so doing the control system must satisfy the requirements relating to the performance of the system such as the desired response, desired control effort etc. and also its implementation. The performance index is a mathematical model of the performance requirements. Many performance indices have been proposed in the literature. The quadratic performance index in the output and control vectors minimizes the error in the output variables and the control effort. Also it results in a linear control law which is easy to implement on practical systems.

The performance criterion is thus taken as[21]

$$J = \frac{1}{2} \int_0^{t_f} (Y^T Q Y + u^T R u)\, dt \tag{4.1}$$

where Q a positive semidefinite matrix, is the weightage associated with the output variables and R a positive definite matrix, is the weightage associated with the

control variables. A unique set of weighting factors to satisfy the prescribed design specifications generally does not exist. However, the lack of uniqueness of the weighting factors does introduce a flexibility which makes selection of the performance index simpler. Depending upon the relative importance of the state or output vector and the control variables, the Q and R matrices are chosen to reflect the desired closed loop system performance. The constraints on the control and output variables can be indirectly taken into account by assigning suitable penalties to the constrained variables. The Q and R matrices themselves can be chosen to reflect these penalties.

The problem posed here is that once the performance criterion is selected, it is required to find an optimal control law which minimizes the performance index subject to the constraints

$$\dot{X} = A X + B u \tag{4.2}$$

$$Y = C X \tag{4.3}$$

The optimal control u can be obtained by the application of Pontryagin's minimum principle or the Hamilton-Jacobi-Bellman theory. The existence of the optimal control requires the prior investigation of the system output controllability as discussed in Appendix B. The optimal control law is obtained[17] (Appendix C) as

$$u = - R^{-1} B^{T} P X \tag{4.4}$$

where P is the solution of matrix Riccati equation

$$\dot{P} + P A + A^T P - P B R^{-1} B^T P + C^T Q C = 0 \qquad (4.5)$$

with $P(t_f) = 0$. For infinite time regulator problem, the solution of algebraic Riccati equation

$$P A + A^T P - P B R^{-1} B^T P + C^T Q C = 0 \qquad (4.6)$$

gives the control law. The optimally controlled system is then given by

$$\dot{X} = (A - B R^{-1} B^T P)X \qquad (4.7)$$

Even an unstable system can be stabilized with the application of optimal control[17].

4.3 SOLUTION OF RICCATI EQUATION

The solutions of Riccati differential and algebraic matrix equations are discussed below.

Infinite Time Problem:

For the solution of algebraic matrix Riccati equation, the method of successive approximation[19] is used. In this method, approximation in control policy space is combined with stability considerations from the second method of Lyapunov. A sequence of suboptimal control functions are then generated which have a monotonic convergence. The initial choice of P is made such that the controlled system given by equation (4.7) is stable. The Riccati equation at the kth iteration is given by

$$P^k A^k + (A^k)^T P^k + Q^k = 0 \qquad (4.8)$$

where

$$A^k = A - B R^{-1} B^T P^{k-1} \tag{4.9}$$

and

$$Q^k = C^T Q C + P^{k-1} B R^{-1} B^T P^{k-1} \tag{4.10}$$

Starting with an initial value for P, the iterations are carried out until the difference between P^{k-1} and P^k elements is within say one percent.

Finite Time Regulator:

The matrix Riccati differential equation is integrated backwards in time from the final time t_f, with $P(t_f) = 0$ till zero time. The matrices are stored at the different time instants and used for the feedback control. It has been shown[35] that the solution matrix P approaches a steady state value for t_f sufficiently large. This value of the steady state matrix P is used as a constant feedback of states for all times from $t = 0$ to $t = t_f$. Also the performance deviation by using this control is not very much from the exact control law[35]. If the time varying control law is used, then the implementation of the feedback law becomes difficult and requires a preprogrammed control.

4.4 PERFORMANCE WITH OPTIMAL REGULATORS

For the system given in Chapter II, the performance index with the following weightage matrices are chosen. The weightage on the output variables $\Delta\delta$, Δw and ΔV_m is selected as $Q = \text{dia}(10,10,10)$ and the weightage matrix on the control

variables ΔE_{fd} and ΔT_1 is selected as $R = \mathrm{dia}(1,1)$. For the infinite time regulator the Riccati matrix is solved by the method of successive approximation and the P-matrix is obtained as

$$P = \begin{bmatrix} 11.424 & 0.0280 & 0.6950 & -0.6370 & 0.6410 & -0.0560 & 0.0580 \\ 0.028 & 0.0610 & -0.0250 & 0.0280 & -0.0250 & 0.0029 & -0.0089 \\ 0.695 & -0.0250 & 2.1400 & -1.9660 & 1.9720 & -0.0134 & 0.0194 \\ -0.637 & 0.0280 & -1.9660 & 1.8100 & -1.8100 & 0.0129 & -0.0187 \\ 0.641 & -0.0250 & 1.9720 & -1.8100 & 1.8200 & -0.0126 & 0.0185 \\ -0.056 & 0.0029 & -0.0134 & 0.0129 & -0.0126 & 0.0240 & -0.0187 \\ 0.058 & -0.0089 & 0.0194 & -0.0187 & 0.0185 & -0.0187 & 0.0178 \end{bmatrix}$$

For the finite time regulator problem, the finite time is taken as $t_f = 5$ seconds. The matrix differential equation is solved backwards in time, giving a steady state solution as

$$P = \begin{bmatrix} 11.436 & 0.0286 & -0.7340 & -0.6720 & 0.6760 & -0.0560 & 0.0590 \\ 0.0286 & 0.0608 & -0.0249 & 0.0283 & -0.0251 & 0.0029 & -0.0089 \\ 0.734 & -0.0249 & 2.1414 & -1.9674 & 1.9728 & -0.0144 & 0.0208 \\ -0.672 & 0.0283 & -1.9674 & 1.8122 & -1.8134 & 0.0139 & -0.0199 \\ 0.676 & -0.0251 & -1.9728 & -1.8134 & 1.8179 & -0.0135 & 0.0197 \\ -0.056 & 0.0029 & -0.0144 & 0.0139 & -0.0135 & 0.0240 & -0.0187 \\ 0.059 & -0.0089 & 0.0208 & -0.0199 & 0.0197 & -0.0187 & 0.0178 \end{bmatrix}$$

The controlled system with the optimal regulator is simulated on IBM 7044 digital computer for the dynamic performance when there is an initial type disturbance in

the field current. The fourth order Runge-Kutta integration method is used to solve the equations. The response of the optimally controlled system is shown in Figure 4.1 for the infinite time regulator and in Figure 4.2 for the finite time regulator. The comparison of Figures 4.1 and 4.2 with the conventional voltage regulator and speed governor response given in Figure 2.7 reveals the superiority of the optimal regulators. The response decays exponentially with no overshoot and oscillations and takes much smaller time to settle down.

4.5 RESPONSE OF THE SYSTEM FOR LARGE DISTURBANCES

The design of the optimal regulators for the single machine system is repeated at different operating conditions. The average value of the performance index is shown[26] to be

$$\hat{J} = \operatorname{tr}(P) \qquad (4.11)$$

The values of $\hat{J}$ are tabulated below for different rotor angles.

δ_0	17.70°	26.3°	58.0°	77.0°	87.0°
$\hat{J}$	16.99	17.3	17.4	16.77	16.12

From the above results, it can be concluded that the performance value with linear time invariant feedback control obtained for one particular operating condition almost remains optimal over different load conditions. This ensures the adaptability of this control even for large disturbances. To prove this point, the performance

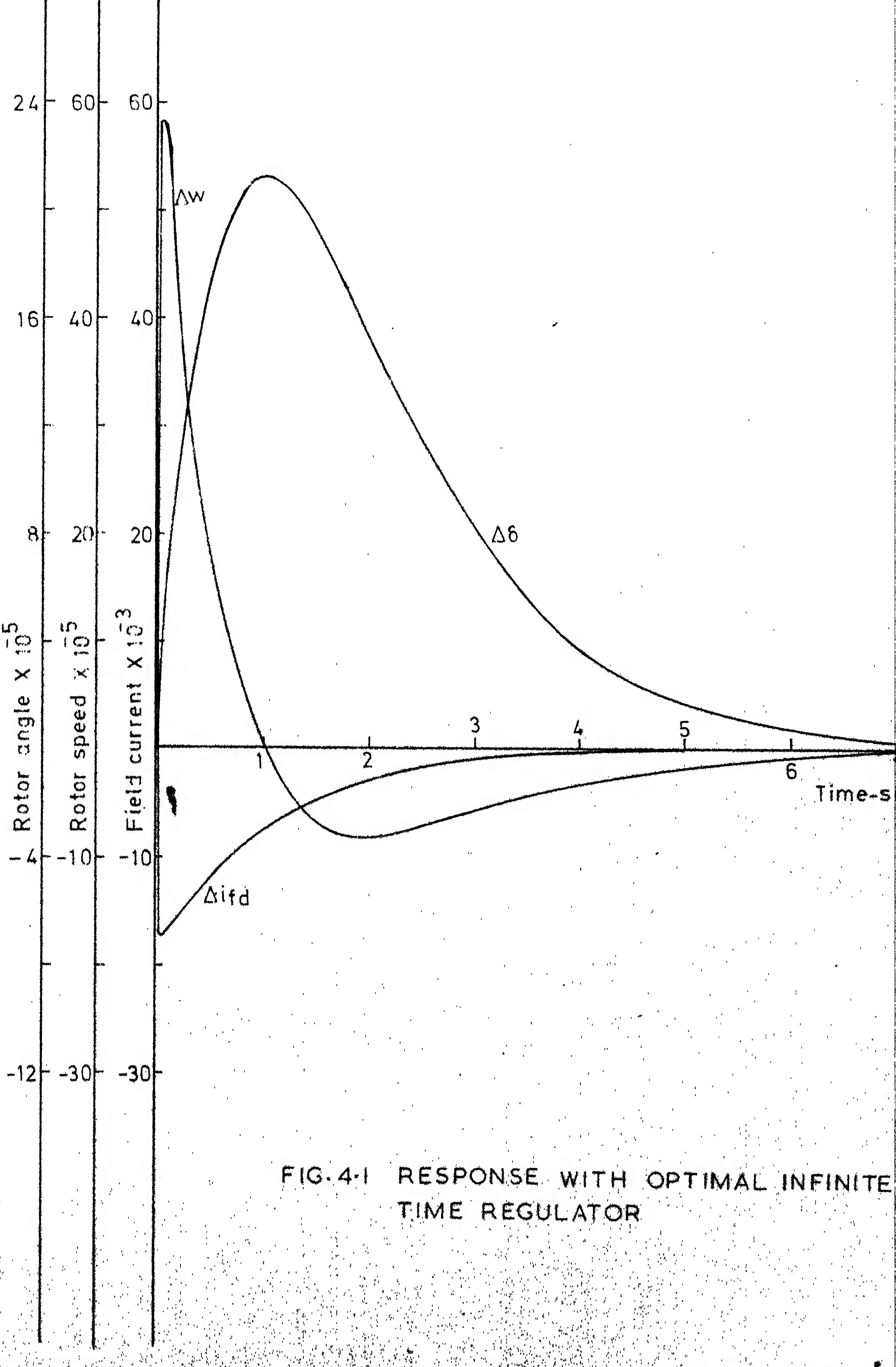

FIG. 4.1 RESPONSE WITH OPTIMAL INFINITE TIME REGULATOR

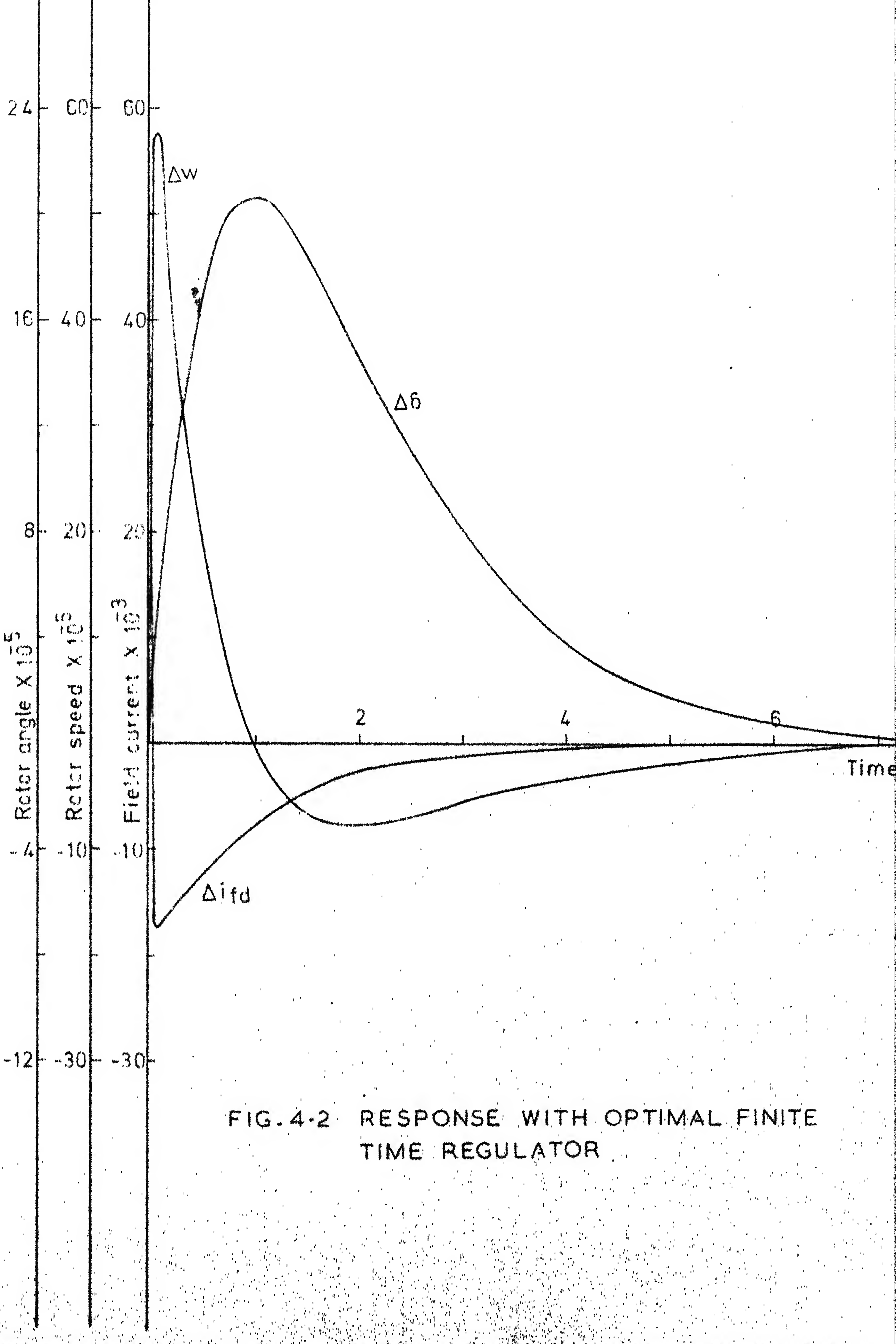

FIG. 4.2 RESPONSE WITH OPTIMAL FINITE TIME REGULATOR

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of the system with optimal regulators is investigated when the second transmission line in Figure 2.1 is reclosed. The system is originally operating with one line in service having the following operating conditions :

$$\delta_o = 28.7^\circ \qquad w_o = 314.0 \qquad i_{fdo} = 2.56 \qquad i_{do} = 0.91$$

$$i_{qo} = 0.4146 \quad i_{kdo} = 0 \quad i_{kqo} = 0 \quad v_{do} = 0.323 \quad v_{qo} = 1.465$$

When the second line is reclosed the problem is to transfer the system state X from $X_1(o)$ to $X_2(t)$. The responses of the system for this state transfer with optimal regulators computed at the post disturbance steady state operating point and with conventional regulators discussed in Chapter II are shown in Figures 4.3 and 4.4 respectively. From these responses, it is seen that the optimal regulator is better than conventional regulators. Thus the optimal regulator calculated at a particular operating point can be used for different load conditions with less performance deterioration.

4.6 CONCLUSION

An optimal output regulator for the system considered is obtained at different operating conditions. The superiority of the optimal integrated control is established by comparison of responses of optimal and conventional regulators. The performance of the system is investigated for large disturbances. The optimal control law requires the entire state vector for the purpose of feedback.

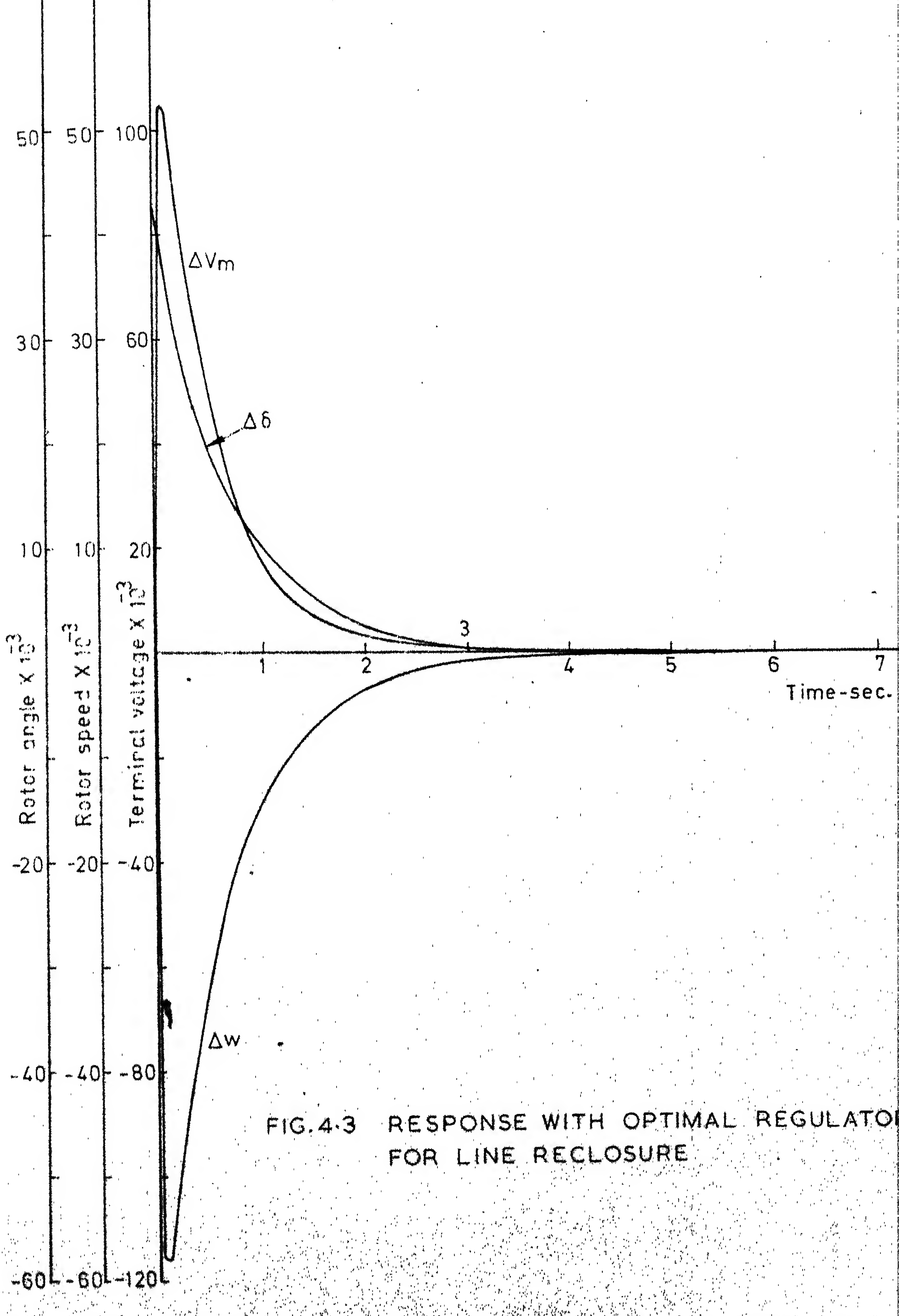


FIG. 4·3 RESPONSE WITH OPTIMAL REGULATOR FOR LINE RECLOSURE

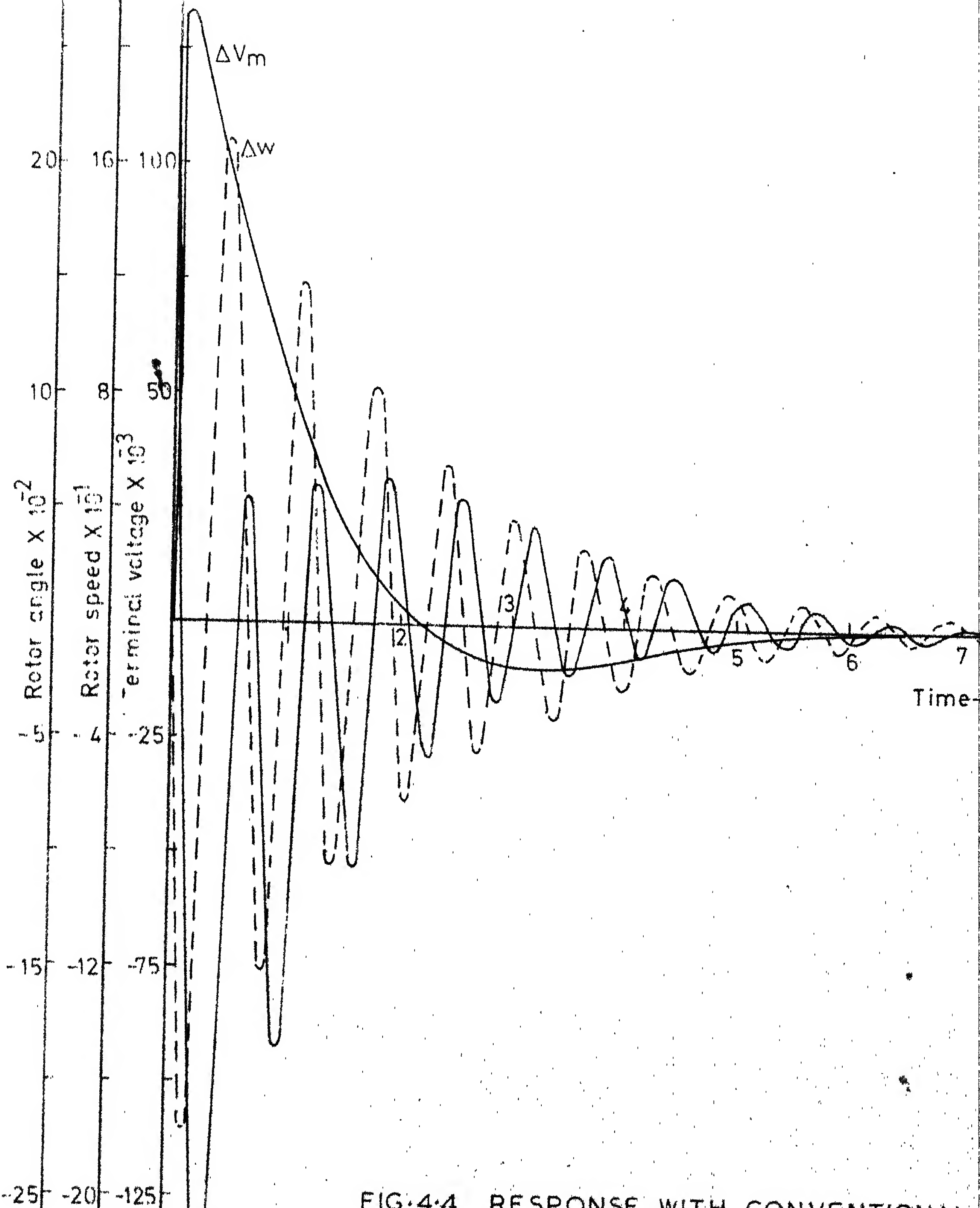


FIG.4.4 RESPONSE WITH CONVENTIONAL REGULATORS FOR LINE RECLOSU

CHAPTER V

A DYNAMIC OBSERVER FOR SYNCHRONOUS MACHINE

5.1 INTRODUCTION

The optimal regulator designed in the last chapter calls for the direct measurement of entire state vector. This is impractical in many problems because the state variables chosen need not necessarily correspond to physically measurable quantities. In some cases the measurement of some of the variables may be forbidden. Even if it is possible, it may not be economical to do so in many cases. As mentioned earlier it may not be possible to know their post fault steady state values to compute the deviations. Therefore it is necessary to reconstruct the states by employing either a Kalman-Bucy filter or a Luenberger type observer, from the measurements of only a few output variables.

In this chapter a dynamic observer of the Luenberger type is discussed which reconstructs the state vector from the available output measurements. The performance of the optimally controlled system obtained in Chapter IV, cascaded with the compatible observer is then obtained. The transfer function matrix relating the input and output of the system is also derived.

It is shown[22] that for a linear, time invariant, finite dimensional, dynamic observable system, it is always possible to obtain a compatible observer. A compatible observer is one whose output equals the state of the system to within an exponentially decaying error.

5.2 DYNAMIC OBSERVER

The problem is that given the linear time invariant system

$$\dot{X} = A X + B u \tag{5.1}$$

$$Y = C X \tag{5.2}$$

it is required to reconstruct the unavailable state variables. The system has to be both output controllable and observable as discussed in Appendix B. For the linear system described by equations (5.1) and (5.2), the observer is defined[22] as

$$\dot{Z} = F Z + G Y + H u \tag{5.3}$$

$$\hat{X} = L_1 Y + L_2 Z \tag{5.4}$$

where $\hat{X}$ is the reconstructed state vector. The observer state Z is p-dimensional where $p = n-m$, n being the system order and m being the number of available outputs. For a choice[22] of

$$H = T B \tag{5.5}$$

where T is the solution of

$$T A - F T = G C \tag{5.6}$$

the observer and system states are related by

$$Z = T X + e \tag{5.7}$$

where e is the error vector between the observed and actual state vectors and is given by

$$e = \exp(F t)\, e(o) \tag{5.8}$$

and e(o) is the error at at t = o. The reconstructed state is given by

$$\hat{X} = L(WX + \hat{e}) \tag{5.9}$$

where $\hat{e}$ is defined as

$$\hat{e} = \begin{bmatrix} o_m \\ e \end{bmatrix} \tag{5.10}$$

with o_m as an m-dimensional null vector. L and W are the partitioned matrices given by

$$L = (L_1 \vdots L_2) \tag{5.11}$$

$$W = \begin{bmatrix} C \\ \cdots \\ T \end{bmatrix} \tag{5.12}$$

In the above treatment it is assumed that F is a stable matrix and F does not have any eigen value in common with that of system matrix A. In order to completely specify the observer dynamics all the above discussed matrices are to be determined. The matrices F and G are chosen such that the pair (F, G) is controllable. The matrix T is obtained in a straight forward manner[22] as

$$T = -\phi^{-1}(F)\, R\, \Omega\, S \qquad (5.13)$$

where

$$R = (G \vdots F G \vdots F^2 G \vdots \ldots\ldots F^{n-1} G) \qquad (5.14)$$

is the controllability matrix of the observer system,

$$S = \left[C^T \vdots A^T C^T \vdots (A^T)^2 C^T \vdots \ldots\ldots (A^T)^{n-1} C^T\right]^T \qquad \ldots (5.15)$$

is the observability matrix of the system given by equations (5.1) and (5.2),

$$\phi(\lambda) = \sum_{i=0}^{n} \alpha_i \lambda^i \quad \text{with } \alpha_n = 1 \qquad (5.16)$$

is the characteristic polynomial of the system matrix A and α_i's are the coefficients of its characteristic equation and

$$\Omega = \begin{bmatrix} \alpha_1 I_m & \alpha_2 I_m & \ldots\ldots & \alpha_{n-1} I_m & \alpha_n I_m \\ \alpha_2 I_m & \alpha_3 I_m & \ldots\ldots & \alpha_n I_m & o_m \\ \ldots & \ldots & \ldots\ldots & \ldots & \ldots \\ \alpha_{n-1} I_m & \alpha_n I_m & \ldots\ldots & o_m & o_m \\ \alpha_n I_m & o_m & \ldots\ldots & o_m & o_m \end{bmatrix} \qquad (5.17)$$

I_m and o_m are mxm identity and null matrices respectively. It has been shown[22] that the observer is compatible if the matrix V is nonsingular.

5.3 SYNCHRONOUS MACHINE OBSERVER

For the system shown in Figure 2.1, it is required to reconstruct the seven state variables from three output variables namely the rotor angle, rotor speed and machine terminal voltage. As discussed in the last section, the observer order becomes four. For the operating point considered in Chapter II with rotor angle of 26.3°, the matrices A, B and C are already given. The system is found to be both output controllable and observable using equations (B.4) and (B.5).

The observer matrices F and G are selected as[23]

$$F = \begin{bmatrix} -20 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & -20 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

The matrices $\emptyset(F)$, R, Ω and S are calculated using equations (5.14) to (5.17). Then the matrix T is obtained from equation (5.13) and the W-matrix is constructed using the matrices C and T. The submatrices L_1 and L_2 are obtained by assuming[22] that $L = W^{-1}$. The H-matrix is

calculated using equation (5.5). Thus all the observer matrices are obtained and are given by

$$T = \begin{bmatrix} 0.0484 & -0.0025 & -0.0061 & 0.0052 & -0.0049 & -0.0119 & 0.0159 \\ 0.0315 & 0.0491 & 0.1226 & -0.1048 & 0.0989 & 0.2376 & -0.3189 \\ 0.0079 & -0.0027 & 0.0268 & -0.0249 & -0.0234 & -0.0068 & 0.0116 \\ 0.0079 & -0.0027 & 0.0268 & -0.0249 & -0.0234 & -0.0068 & 0.0116 \end{bmatrix}$$

$$H = \begin{bmatrix} -0.0029 & 0.0597 & 0.0093 & 0.0093 \\ -0.1277 & 2.5550 & -0.0139 & -0.0139 \end{bmatrix}^T$$

$$L_1 = \begin{bmatrix} 1 & 0 & 4.2175 & 526.90 & 264.90 & 4231.60 & 3063.55 \\ 0 & 1 & -0.0068 & -0.0400 & -0.0689 & -0.3218 & -0.0968 \\ 0 & 0 & 0.3068 & 4.7506 & 3.8470 & 11.558 & 8.3789 \end{bmatrix}^T$$

$$L_2 = \begin{bmatrix} 0 & 0 & -86.52 & -10530.0 & -5289.0 & -3.496E9 & -61270 \\ 0 & 0 & -4.503 & -525.40 & -262.40 & -84630 & -3064 \\ 0 & 0 & -2.896E9 & 2.801E8 & 3.496E9 & -4224 & 1.54E9 \\ 0 & 0 & 2.896E9 & -2.801E8 & -3.496E9 & 4224 & -1.54E9 \end{bmatrix}^T$$

In the matrix L_2, E* means 10^*. Once these matrices are determined the observer dynamics are completely specified.

5.4 PERFORMANCE WITH DYNAMIC OBSERVER

The observer constructed in the previous section is cascaded with the optimal infinite time regulator determined in Chapter IV. The response of the system for a disturbance of 0.05 p.u. in the field current is obtained by simulating

the cascaded system. The response is shown in Figure 5.1. The responses have more overshoot in the initial portions but decays fast to the steady state values.

The observer matrices are found[23] to be sensitive to the system operating conditions. Hence for each operating point, the observer dynamics are different and therefore the control law is different for the various operating conditions. But this is not the case with all the state variables available for measurements as discussed in the last chapter. The error in the reconstructed state vector decays exponentially depending upon the values of the elements of F-matrix as given by equation (5.10). By choosing large negative eigen values for F, the error can be reduced; but this introduces large gains in the transfer matrix between input and output. By increasing the weightages on the output variables the system can be brought to equilibrium state in a quicker time at the expense of large inputs. Hence a compromise must be sought between these two.

5.5 TRANSFER MATRIX

The transfer matrix[23] relating the output and input are derived by assuming that all the initial conditions are zero. The various equations are written in Laplace transformed variables as follows :

From equation (5.3)

$$Z(s) = (sI - F)^{-1} [G\ Y(s) + H\ U(s)] \tag{5.18}$$

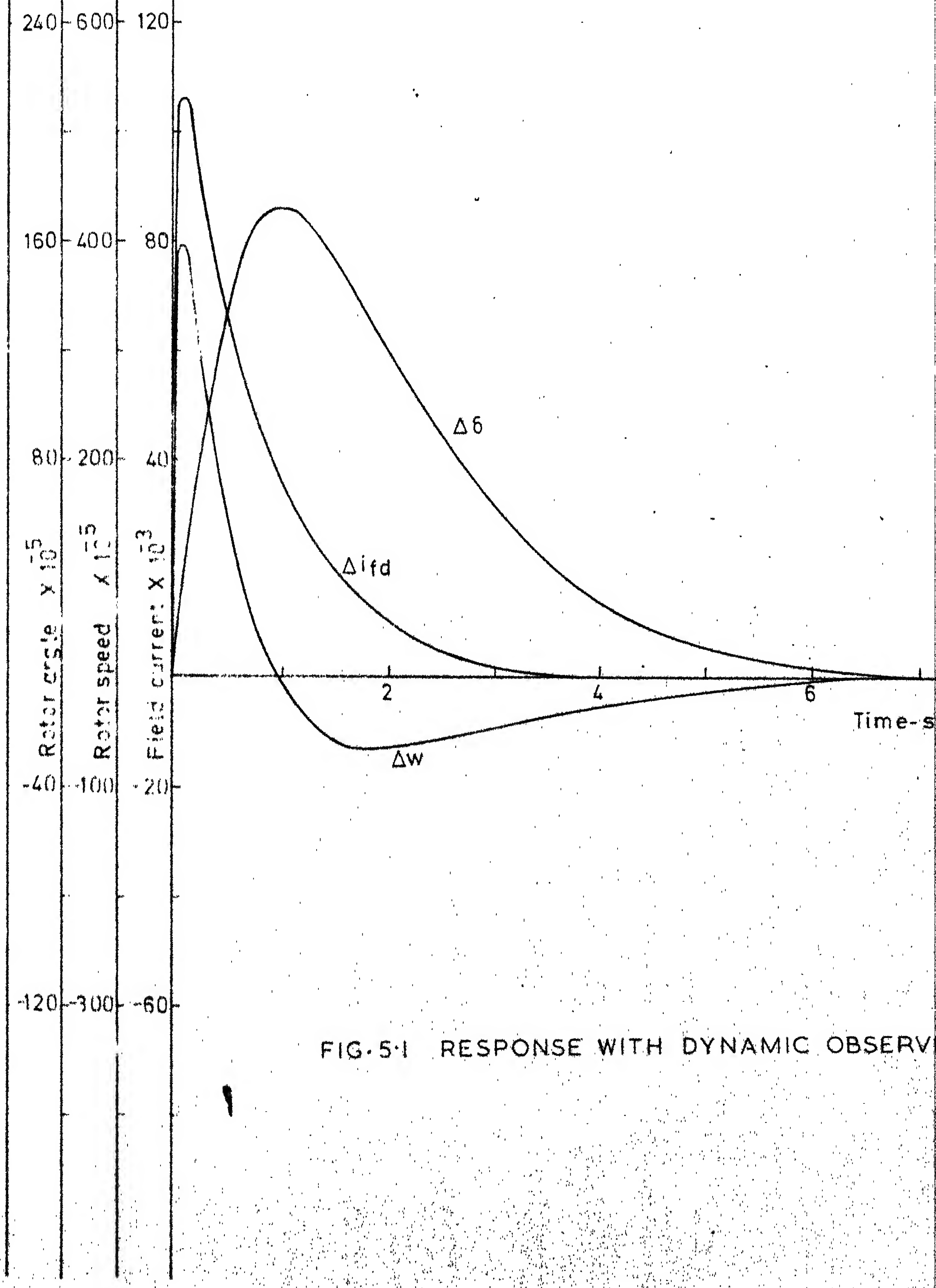


FIG. 5·1 RESPONSE WITH DYNAMIC OBSERV

From equation (5.4)

$$\hat{X}(s) = L_1\ Y(s) + L_2\ Z(s) \tag{5.19}$$

From equation (4.4)

$$U(s) = F\ X(s) \tag{5.20}$$

where

$$F = -\ R^{-1}\ B^T\ P \tag{5.21}$$

Assuming that $X(s) = \hat{X}(s)$, the following equation is obtained

$$U(s) = F\,[L_1\ Y(s) + L_2\ Z(s)] \tag{5.22}$$

Substituting for $Z(s)$ from equation (5.18), the equation (5.22) reduces to

$$U(s) = F\,[L_1 + L_2\ K(s)\ G]\ Y(s) + F\ L_2\ K(s)\ H\ U(s) \qquad \ldots \tag{5.23}$$

where

$$K(s) = (sI - F)^{-1} \tag{5.24}$$

From equation (5.23), $U(s)$ is obtained as

$$U(s) = [I - F\ L_2\ K(s)\ H]^{-1}\ F\,[L_1 + L_2\ K(s)\ G]\,Y(s) \qquad \ldots \tag{5.25}$$

Thus the transfer function matrix $N(s)$ in the relation

$$U(s) = N(s)\ Y(s) \tag{5.26}$$

for the operating conditions considered, is obtained using equation (5.25) as

$$N(s) = \frac{1}{D(s)} \begin{bmatrix} 145.5s^2+2205s+38 & 0.02s^2-s-2096 & 0.27s^2+9.5s+16.5 \\ 355s^2+7140s-777 & (-3.2s^2-462.5s-7311) & 0.63s^2+73s+1220 \end{bmatrix}$$

where

$$D(s) = s^2 + 37.85s + 354.32$$

Thus the i,jth element in the transfer matrix N(s) refers to the transfer function between the jth output and ith input and this can be practically implemented. However, this control gives optimal response only for the specified operating point. The transfer function matrix N(s) is different for the different operating conditions.

5.6 CONCLUSION

A compatible dynamic observer is obtained for the synchronous machine system to reconstruct the unavailable states. The effect of observer dynamics is that an exponentially decaying error is introduced in the estimated value of the states. The observer dynamics are very much sensitive to the operating conditions. Thus for each operating point the observer dynamics have to be modified. The input output transfer functions are derived which indicates that time varying functions are introduced in the feedback paths. The gains of these feedback paths are different for different operating conditions. The response of the cascaded system is obtained and compared with complete state feedback system response.

Thus even though the difficulty in the measurement of post fault steady state vector is avoided by a

choice of measurable output variables whose steady state values for all operating conditions can be obtained apriori, the major defect of the control scheme proposed in this chapter is that the transfer function matrix is different for different operating conditions. This makes the optimal control law useless for large perturbations unless a pre-programmed controller is used. This of course will be practically difficult to implement. To overcome this difficulty suboptimal controls which give rise to reasonably good response and also which are linear and time invariant, are considered in the following chapter.

CHAPTER VI

SUBOPTIMAL CONTROL OF SYNCHRONOUS MACHINE

6.1 INTRODUCTION

In recent years, the concept of state space and the use of Pontryagin's minimum principle have resulted in analytical techniques for the synthesis of optimal regulators for multivariable systems. The resulting optimal control law calls for a complete measurement of the state vector. The state variables being mathematical variables introduced for the convenience need not necessarily correspond to physically measurable quantities. Thus one must consider the problem of controlling a physical system with only some of the state variables available for feedback.

Often it is suggested that the unavailable state variables can be reconstructed via a Kalman-Bucy filter or a state Reconstructer[24]. But this introduces transfer functions[28] in the feedback paths. It is quite impractical to reconstruct the state variables in large multivariable systems such as those which occur in interconnected power systems, chemical process control etc. Further it is shown that the resulting gains and time constants in the feedback path depend on the operating conditions. Thus it would be better if the control variables are chosen by

a linear combination of the available output variables instead of employing an observer. This may result in deterioration of the performance of the system. However if the performance is reasonably close to the optimal one then it will be better to go in for this linear time invariant control law.

In this chapter, a suboptimal control technique is outlined where in it is not necessary to reconstruct the unavailable state variables. Thus the merit of this approach lies in the fact that a linear time invariant control law is obtained which is easy to implement. The performance of the system with suboptimal control is determined. The feasibility of using a particular control law over wide range of operating conditions is then discussed.

6.2 STATEMENT OF THE PROBLEM

The problem posed here is the following: Given a linear time invariant dynamic system

$$\dot{X} = A X + B u \tag{6.1}$$

$$Y = C X \tag{6.2}$$

it is required to find an optimal control law which minimizes the quadratic performance index,

$$J = \frac{1}{2} \int_0^{\infty} (X^T Q X + u^T R u) dt \tag{6.3}$$

In addition, the control law is constrained to be a linear function of the output variables as

$$u = F\,Y \tag{6.4}$$

If all the state variables are available for feedback (i.e. if C^{-1} exists) then the optimal control law for the state regulator discussed above is given by[17]

$$u = \Gamma\ X \tag{6.5}$$

where

$$F^* = -R^{-1}B^T P \tag{6.6}$$

and P is the positive definite solution of

$$P A + {}^T P - P B R^{-1} B^T P + Q = 0 \tag{6.7}$$

The optimal value of the performance index is shown[17] to be

$$J = X(o)^T P\, X(o) \tag{6.8}$$

Suboptimal Control:

The control law is constrained to be a function of output variables. Then using equation (6.2), the control law becomes

$$u = F\,C\,X \tag{6.9}$$

Then the performance index is given by

$$J = \frac{1}{2}\int_0^{\infty} X^T(Q + C^T F^T R F C)X\,dt \tag{6.10}$$

and the closed loop system becomes

$$\dot{X} = (A + B F C)X, \quad X(o) = X_o \tag{6.11}$$

Now, the problem is to find the elements of the control matrix F, which minimizes the performance index J. Substituting the solution of state vector obtained from equation (6.11) in equation (6.10), the performance index is obtained as

$$J = \frac{1}{2} X(o)^T \{ \int_0^\infty \phi^T(t) \; [Q + C^T F^T R F C] \phi(t) dt \} X(o) \quad \ldots (6.12)$$

where

$$\phi(t) = \exp (A + B F C)t \qquad (6.13)$$

is the fundamental matrix for the system given by equation (6.11). From equation (6.12), it is seen that the performance index is a function of both the control matrix F and the initial state X(o). To avoid the dependence of J on X(o), the initial state can be treated as a random vector uniformly distributed over the surface of unit sphere[26,27]. Then the average value of the performance index is given by[26]

$$\hat{J} = \frac{1}{2n} \int_0^\infty tr \; [\phi^T(t) \; (Q + C^T F^T R F C) \; \phi(t)] \; dt \quad (6.14)$$

Thus the problem reduces to the determination of a suboptimal control matrix F, which minimizes $\hat{J}$ subject to the constraint equation (6.11).

Using the trace minimization procedure, the solution of the suboptimal control problem is shown to be[26]

$$F = - R^{-1} B^T K L C^T (C L C^T)^{-1} \qquad (6.15)$$

where K is the positive semidefinite solution of

$$K A_1 + A_1^T K + Q + C^T F^T R F C = 0 \tag{6.16}$$

L is the positive definite solution of

$$L A_1^T + A_1 L + I = 0 \tag{6.17}$$

and A_1 is a stable matrix given by

$$A_1 = (A + B F C) \tag{6.18}$$

The suboptimal matrix F is obtained by an iterative algorithm[26] using equations (6.15) to (6.18). An initial value of F is chosen such that the matrix A_1 is stable. Then matrices K and L are solved and then a new value of F is obtained using equation (6.15). The process is repeated until the difference between successive values of the elements of the matrix F is within one percent.

If C^{-1} exists, then the control law obtained is the same as Kalman's optimal regulator. The expected value of the performance index can be shown to be[26]

$$\bar{J} = tr(K). \tag{6.19}$$

For the complete state feedback case, the average value of performance index is

$$\hat{J} = tr(P) \tag{6.20}$$

It is also shown that[26]

$$tr(P) < tr(K) \tag{6.21}$$

Hence the control law obtained using equation (6.8) is not optimal but it will be used as a suboptimal control law.

6.3 SUBOPTIMAL REGULATOR FOR THE SYSTEM

The following operating conditions are obtained when the machine is delivering rated KVA at unity power factor to the infinite bus, using the phasor diagram given in Figure 2.3 :

$\delta_o = 16.0°$ $\quad w_o = 314.0$ $\quad i_{fdo} = 1.82$ $\quad i_{do} = 0.72$

$i_{qo} = 0.694$ $\quad i_{kdo} = 0$ $\quad i_{kqo} = 0$

$V_{mo} = 1.092$ $\quad v_{do} = 0.548$ $\quad v_{qo} = 0.945$

The system matrices A, B and C are calculated for the above conditions and are given by

$$A = \begin{bmatrix} 0.0 & 1.000 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.0 & -0.167 & -36.14 & 14.56 & -36.14 & -79.56 & 22.50 \\ -419.0 & 1.067 & -2.13 & -36.23 & -24.15 & 664.23 & -362.30 \\ -879.9 & 2.240 & -0.66 & -76.08 & -12.07 & 1395.00 & -760.85 \\ -419.0 & 1.067 & 1.33 & -36.23 & -38.64 & 664.23 & -362.30 \\ 696.0 & 2.720 & 897.14 & -1345.70 & 897.14 & -53.83 & -26.91 \\ 484.5 & 2.040 & 672.86 & -1009.28 & 672.86 & -40.37 & -35.88 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0 & 0.0 & 2.125 & 0.664 & -1.33 & 0.0 & 0.0 \\ 0.0 & 52.08 & 0.00 & 0.00 & 0.00 & 0.0 & 0.0 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ -0.275 & 0.00 & 0.00 & 0.285 & 0.00 & -0.107 & 0.00 \end{bmatrix}$$

The weightage matrices Q and R for the state and control variables respectively are chosen as identity matrices and the optimal feedback Riccati matrix for the infinite time state regulator is obtained by the method of successive approximation as[28]

$$P = \begin{bmatrix} 2.270 & 0.009 & 1.5800 & -1.4500 & 1.4500 & -0.078 & 0.084 \\ 0.009 & 0.079 & -0.0016 & 0.0035 & -0.0035 & -0.003 & -0.006 \\ 1.580 & -0.0016 & 4.3800 & -4.0000 & 3.9900 & -0.009 & 0.015 \\ -1.450 & 0.0035 & -4.0000 & 3.6800 & -3.6600 & 0.071 & -0.016 \\ 1.450 & -0.0035 & 3.9900 & -3.6600 & 3.6700 & -0.011 & 0.016 \\ -0.078 & 0.0030 & -0.0090 & 0.0110 & -0.0110 & 0.082 & -0.078 \\ 0.084 & -0.0060 & 0.0150 & -0.0160 & 0.0160 & -0.078 & 0.087 \end{bmatrix}$$

The suboptimal control matrix F is obtained by the iterative procedure[26] using equations (6.15) to (6.18) for the above operating conditions as

$$F = \begin{bmatrix} -0.5857 & -0.5000 & 1.4761 \\ 0.9353 & 1.1428 & 0.2071 \end{bmatrix}$$

and the K matrix is given by

$$K = \begin{bmatrix} 2.240 & 0.014 & 1.420 & -1.310 & 1.300 & -0.078 & 0.081 \\ 0.014 & 0.022 & -0.002 & 0.004 & -0.004 & 0.002 & -0.005 \\ 1.420 & -0.002 & 4.48 & -4.090 & 4.08 & 0.028 & -0.009 \\ -1.310 & 0.004 & -4.090 & 3.760 & -3.740 & 0.005 & 0.057 \\ 1.300 & -0.004 & 4.080 & -3.740 & 3.750 & 0.005 & -0.006 \\ -0.078 & 0.002 & 0.008 & 0.005 & 0.005 & 0.087 & -0.081 \\ 0.081 & -0.005 & -0.009 & 0.057 & -0.006 & -0.081 & 0.090 \end{bmatrix}$$

By constraining a positive definite solution for the L-matrix during each stage of computation, the system stability is assured with the suboptimal control.

6.4 PERFORMANCE WITH SUBOPTIMAL CONTROL

The performance of the single machine system with the suboptimal feedback control law is obtained for the operating point considered. The response of the sub-optimal control system for an initial disturbance in the field current of 0.05 p.u. is shown in Figure 6.1 and the response with complete state feedback for the same conditions is shown in Figure 6.2. The average performance index value is more only by 0.23 over the optimal value. The performance deviation, by the use of suboptimal control is very less. The suboptimal design is repeated for different operating conditions. The suboptimal control law obtained for one operating point is used at different operating conditions and the average performance index

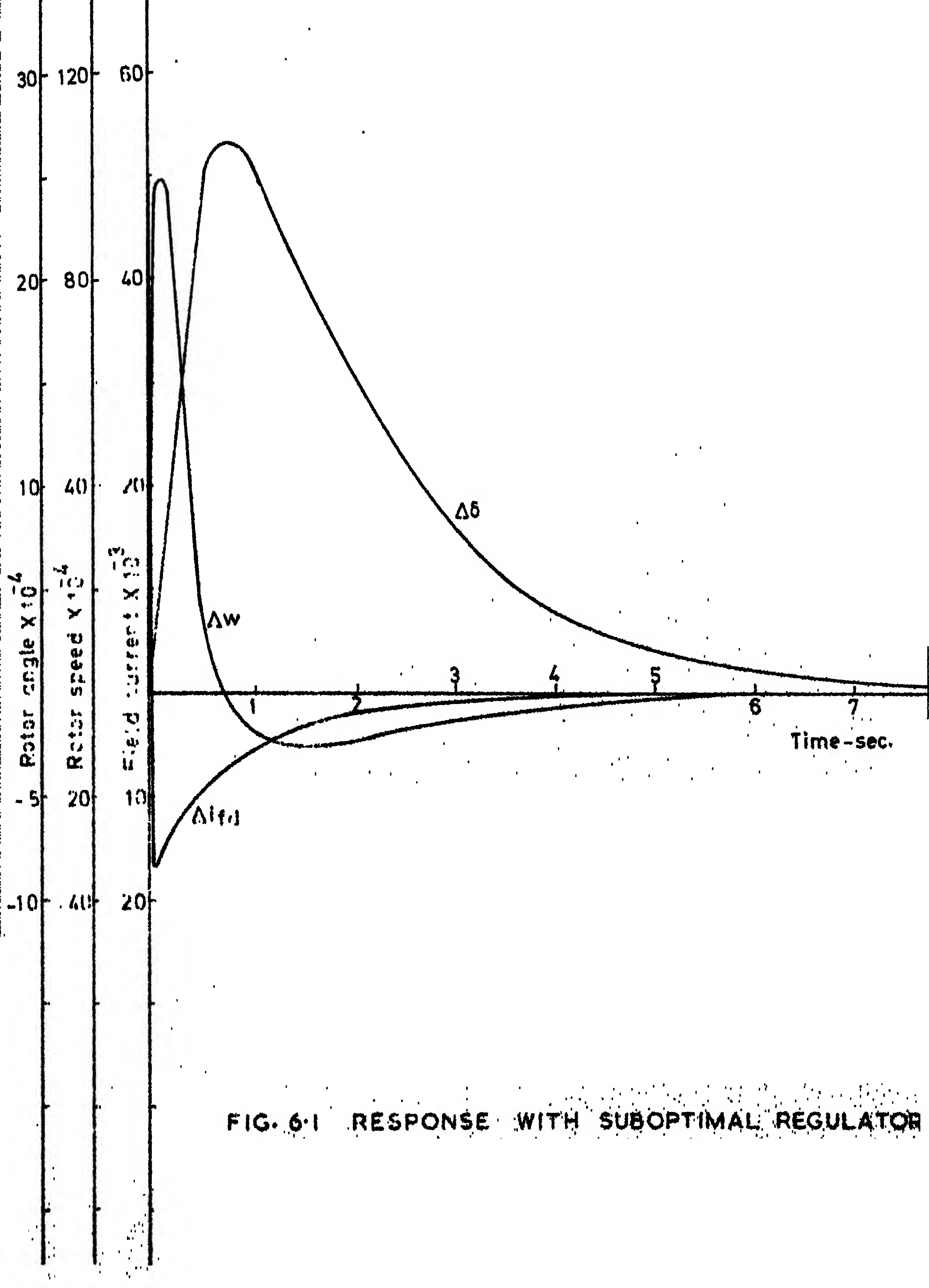


FIG. 6·1 RESPONSE WITH SUBOPTIMAL REGULATOR

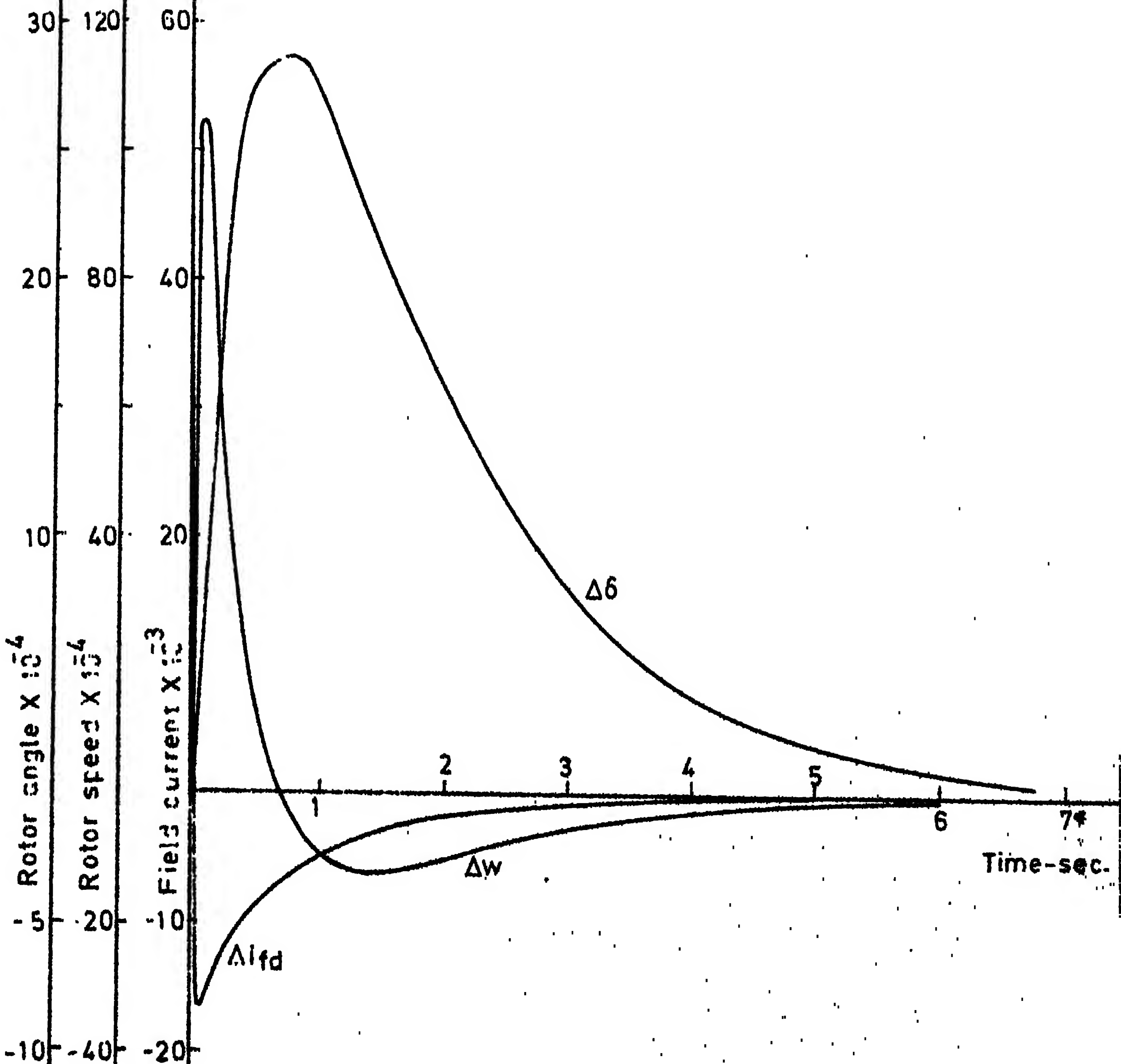


FIG. 6·2 RESPONSE WITH OPTIMAL STATE REGULATOR

values are calculated and are given below.

Rotor Angle	Expected value of J		
	Optimal	Suboptimal	Suboptimal with F at 46°
17.7°	14.73	14.91	15.32
26.3°	14.52	14.54	14.64
46.0°	14.20	14.43	14.43

From the above results it can be concluded that control matrix F calculated at $\delta = 46°$ can be used over a range of load conditions between say $\delta = 15°$ to $\delta = 60°$ without much performance deterioration. This ensures that the suboptimal control law given by equation (6.9) is reasonably good even for large disturbances.

The system performance for a large disturbance due to the line reclosure for the system shown in Figure 2.1 is discussed with the suboptimal control law obtained at the post disturbance steady state conditions. The predisturbance operating conditions are :

$$\delta_o = 51.6° \quad w_o = 314.0 \quad i_{fdo} = 2.004 \quad i_{do} = 0.785$$
$$i_{qo} = 0.62 \quad i_{kdo} = 0 \quad i_{kqo} = 0 \quad v_{do} = 0.49 \quad v_{qo} = 1.154$$

The performance curves are plotted in Figure 6.3 for this system state transfer with suboptimal regulator and in Figure 6.4 for the same conditions with complete state feedback.

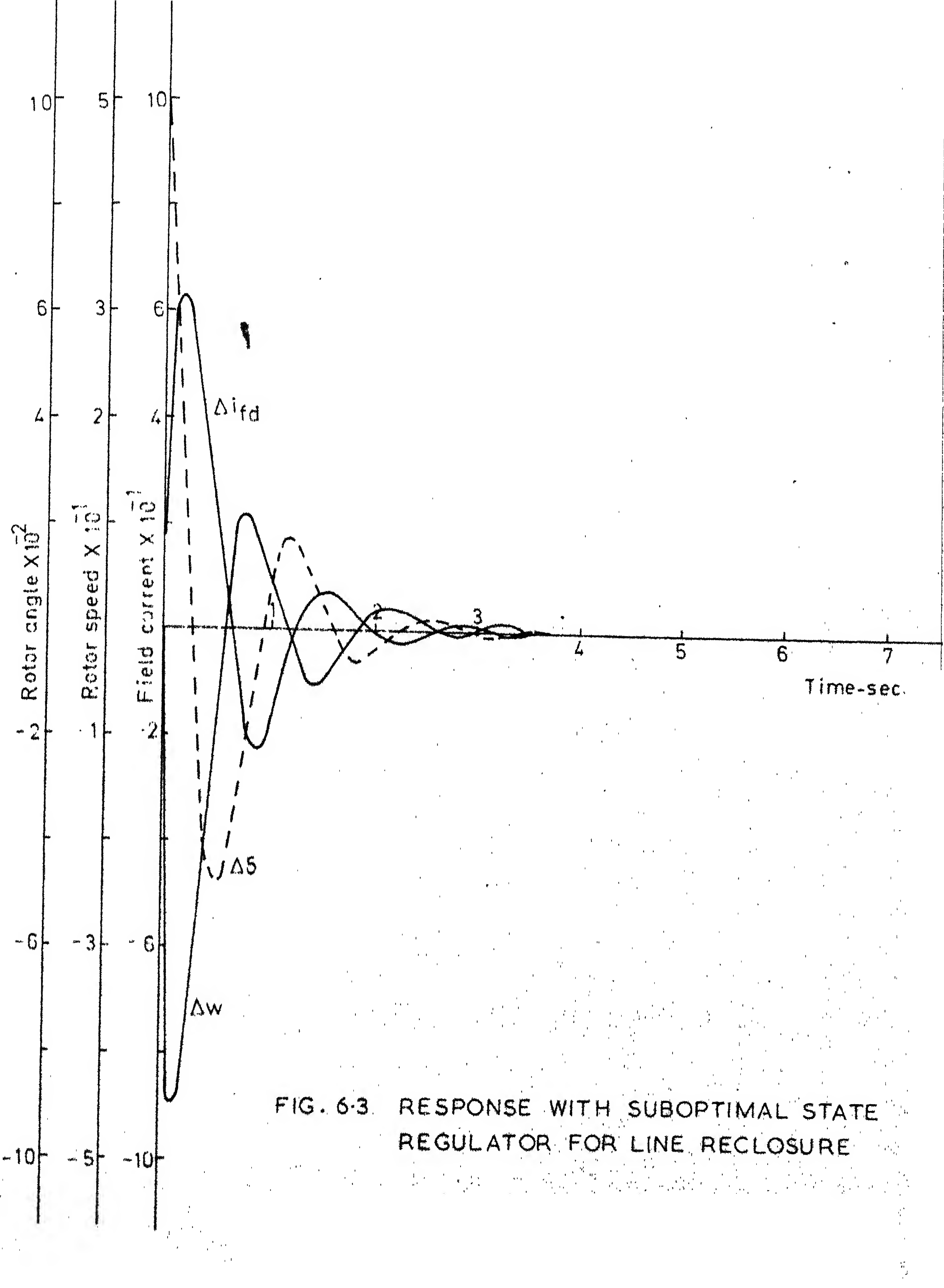


FIG. 6·3 RESPONSE WITH SUBOPTIMAL STATE REGULATOR FOR LINE RECLOSURE

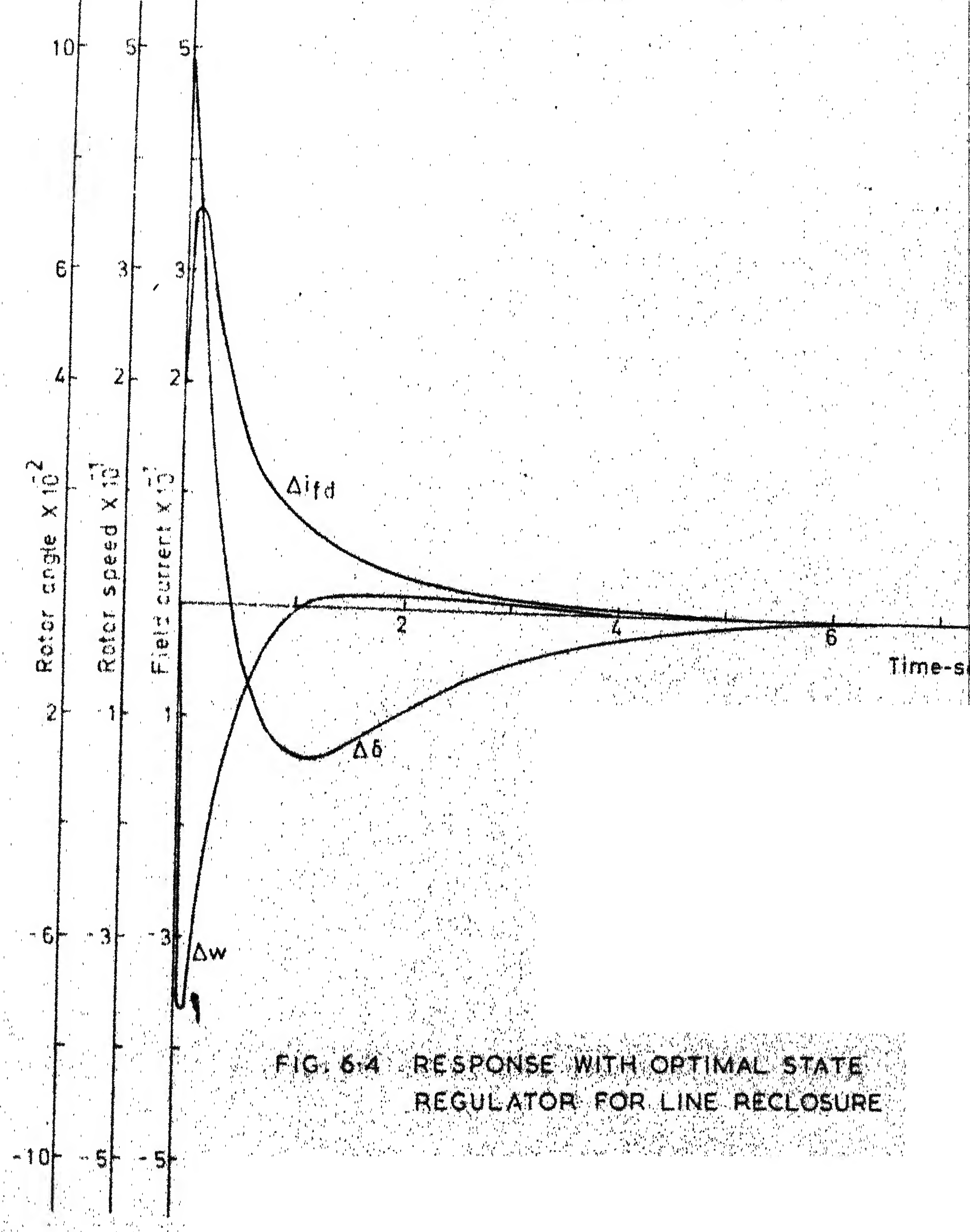


FIG. 6.4 RESPONSE WITH OPTIMAL STATE REGULATOR FOR LINE RECLOSURE

In the above analysis the output variables used for the feedback are $\Delta\delta$, Δw and ΔV_m. The steady state values of Δw and ΔV_m are zero irrespective of the operating conditions and hence there is no difficulty in obtaining the deviations required for feedback. It is necessary to know the post fault steady state power angle to obtain the deviations of δ. This will be difficult to know apriori in large systems. However, if the machine operates at constant load angle and the internal voltage E_1 is varied to deliver different powers (as in the case of d.w.r. machines which will be discussed in the next chapter) this difficulty can be circumvented. As an alternative the output variables can be chosen to be Δw and ΔV_m and the sub-optimal control can be obtained. In this case the measurement of deviations of all the output variables will be easy because their steady state values are zero for all operating conditions.

For the operating point discussed earlier when the machine is delivering rated KVA at unity power factor to the infinite bus, with both lines in service, the suboptimal control law is obtained in terms of the new set of output variables. The feedback control matrix for this case is given by

$$F = \begin{bmatrix} 5.6583 & -9.3192 \\ -27.3824 & 25.7142 \end{bmatrix}$$

The value of the performance index using equation (6.20) is obtained as 19.3941 for this suboptimal control and the deviation from the optimal one is 4.8714.

6.5 CONCLUSION

The suboptimal control of the synchronous machine system is discussed. The control law is constrained to be a linear combination of the output variables, which are available for direct measurements. The feasibility of using a particular suboptimal control law at different operating points is investigated. The performance of the system with the suboptimal control is compared with that of a complete feedback case. The performance of the system with suboptimal control is studied for large system disturbances. The implementation of the suboptimal control law is simple as it requires the measurement of only the available output variables.

CHAPTER VII

STATE SPACE MODEL OF DIVIDED WINDING ROTOR SYNCHRONOUS MACHINE

7.1 INTRODUCTION

In this chapter, a d.w.r. synchronous machine with two field windings displaced by 60°E is considered. The state space model is derived in a form most suitable for the application of optimal regulator design. The response of the uncontrolled system is compared with the system response provided with angle and voltage regulators on the field windings and a conventional speed governor for the control of input torque.

With the growth of large national grids, power is transmitted in bulk over long distances at very high voltages. In recent times the use of high voltage cables is also in the increase. Thus the system has to supply large reactive power. For economic reasons, modern generators are of large capacity and are designed with low short circuit ratios. All these factors add to the problem of maintaining stability especially under leading power factor operations. A number of methods were suggested to overcome the stability problem. One method suggested is the use of synchronous machines with an additional field winding and suitable excitation control systems.

Sopper and Fagg[30] considered a machine with two field windings displaced by 60°E, having angle regulator on one winding and voltage regulator on the other winding. Kapoor et.al.[31] analysed a machine with one winding on the direct axis having fixed control and another winding on the quadrature axis with angle regulator. Ramamurthy et.al.[32] used a machine with two field windings, one in d-axis and another on the q-axis, provided with angle and voltage regulators respectively on them. Krause and Towle[33] considered a machine with two field windings for the synchronous machine damping.

By all these studies, it has been established that such a machine has greater steady state and transient stability limits than the normal machine employing conventional regulators. The limit of reactive power capacity for normal machines at no load can be shown to be[31] V_m^2/x_q. The d-axis regulation has the effect of reducing x_d only. But the q-axis regulation modifies x_q and thereby extends the reactive limit. Even under loaded conditions the reactive limit is more for these machines.

In the conventional synchronous machine, the excitation phasor is rigidly fixed to the rotor structure and displaced by the rotor angle from the synchronously revolving voltage phasor of the power system during steady state operating conditions. By having a second field winding the excitation phasor can be freed from the rotor

structure and this permits the rotor to occupy any other suitable position depending upon the relative magnitudes of the two field winding voltages.

7.2 STATE SPACE MODEL

The schematic diagram of d.w.r. synchronous machine is shown in Figure 7.1 and the equivalent d,q machine in Figure 7.2. The d.w.r. machine is connected to the infinite bus through a double circuit transmission line as shown in Figure 7.3. The general nonlinear system equations describing the performance of the system are given by[30]:

Direct Axis Flux Linkages:

$$\psi_d = x_{atd}\, i_t + x_{ard}\, i_r + x_{ad}\, i_{kd} - x_d\, i_d \tag{7.1}$$

$$\psi_{kd} = x_{atd}\, i_t + x_{ard}\, i_r - x_{ad}\, i_d + x_{kkd}\, i_{kd} \tag{7.2}$$

Quadrature Axis Flux Linkages:

$$\psi_q = x_{atq}\, i_t - x_{arq}\, i_r - x_q\, i_q + x_{aq}\, i_{kq} \tag{7.3}$$

$$\psi_{kq} = x_{atq}\, i_t - x_{arq}\, i_r - x_{aq}\, i_q + x_{kkq}\, i_{kq} \tag{7.4}$$

Torque Winding Flux Linkages:

$$\psi_t = x_t\, i_t + x_{tr}\, i_r - x_{atd}\, i_d - x_{atq}\, i_q + x_{atd}\, i_{kd} + x_{atq}\, i_{kq} \tag{7.5}$$

Reactive Winding Flux Linkages.

$$\psi_r = x_{tr}\, i_t + x_r\, i_r - x_{ard}\, i_d + x_{arq}\, i_q + x_{ard}\, i_{kd} - x_{arq}\, i_{kq} \tag{7.6}$$

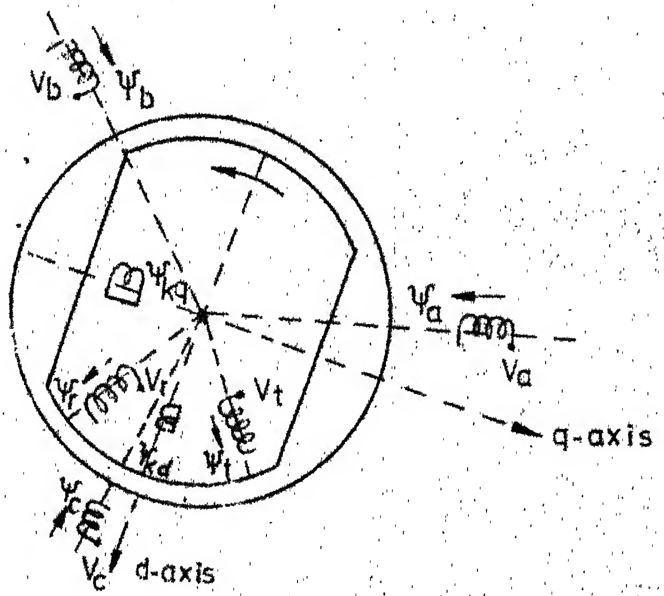


FIG 7.1 SCHEMATIC DIAGRAM OF D.W.R. MACHINE

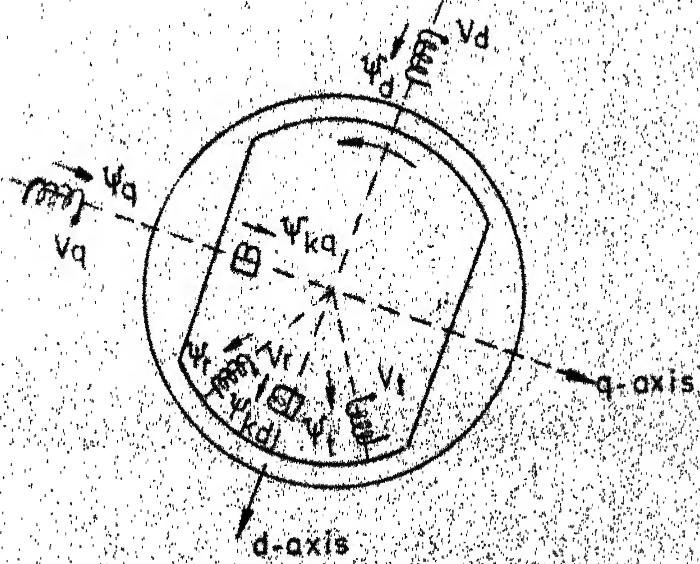


FIG 7.2 EQUIVALENT d,q MACHINE

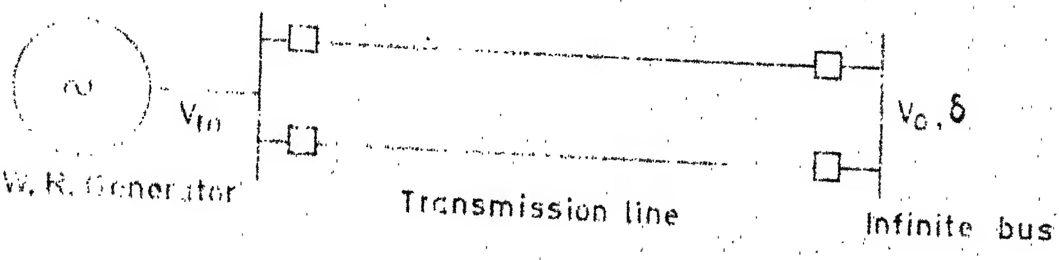


FIG. 7·3 SINGLE MACHINE (D.W.R.) SYSTEM

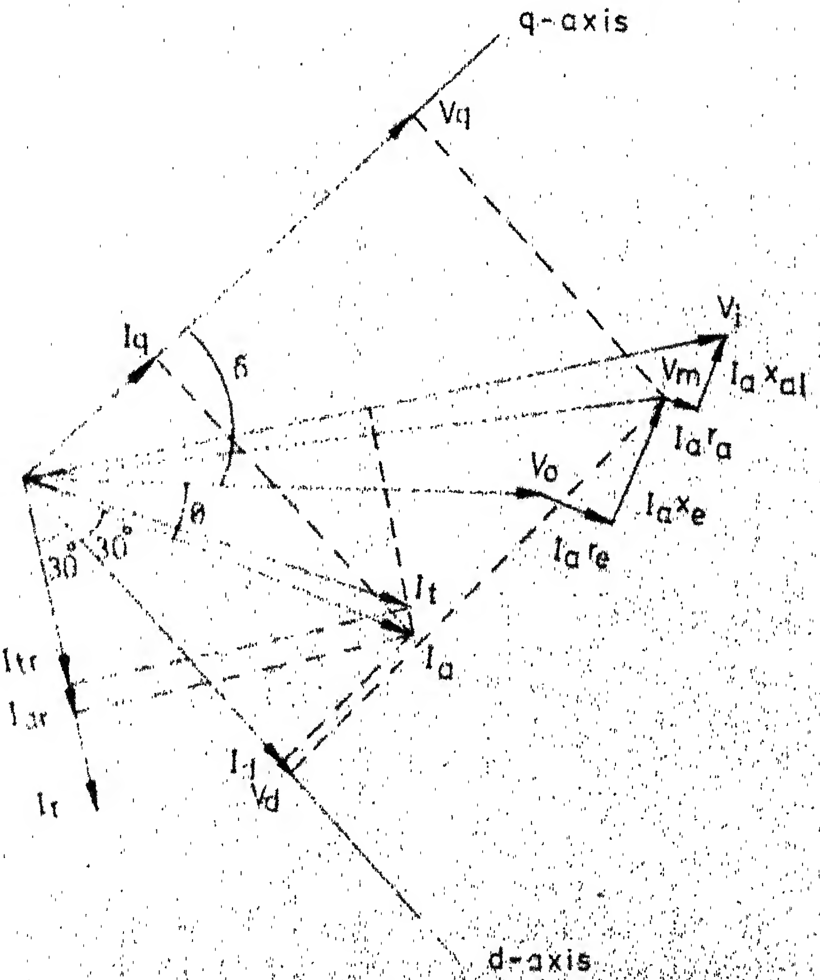


FIG. 7·4 PHASOR DIAGRAM OF D.W.R. GENERATOR

Direct Axis Voltages:

$$v_d = \frac{1}{w_o} p \psi_d - r_a i_d - \frac{w}{w_o} \psi_q \tag{7.7}$$

$$0 = \frac{1}{w_o} p \psi_{kd} + r_{kd} i_{kd} \tag{7.8}$$

Quadrature Axis Voltages:

$$v_q = \frac{1}{w_o} p \psi_q - r_a i_q + \frac{w}{w_o} \psi_d \tag{7.9}$$

$$0 = \frac{1}{w_o} p \psi_{kq} + r_{kq} i_{kq} \tag{7.10}$$

Torque Winding Voltage:

$$v_t = \frac{1}{w_o} p \psi_t + r_t i_t \tag{7.11}$$

Reactive Winding Voltage:

$$v_r = \frac{1}{w_o} p \psi_r + r_r i_r \tag{7.12}$$

Equation of Motion of Rotor:

$$M \frac{d^2 \delta}{dt^2} = T_i - T_e - K_d \frac{d \delta}{dt} \tag{7.13}$$

where

$$T_e = \psi_d i_q - \psi_q i_d \tag{7.14}$$

is the electrical torque in the air gap.

Generator Terminal Voltage Condition:

$$V_m^2 = v_d^2 + v_q^2 \tag{7.15}$$

Transmission Line Equations:

$$v_d = V_o \operatorname{Sin} \delta + r_e i_d - x_e i_q \tag{7.16}$$

$$v_q = V_o \operatorname{Cos} \delta + r_e i_q + x_e i_d \tag{7.17}$$

Substituting for the flux linkages from equations (7.1) to (7.6) in the other performance equations, they can be put in the vector form as

$$E \dot{X} = f(X) + D u \tag{7.18}$$

where the state vector X is chosen as

$$X = (\delta, w, i_t, i_r, i_d, i_q, i_{kd}, i_{kq})^T \tag{7.19}$$

and the control vector u is given by

$$u = (v_t, v_r, T_i)^T \tag{7.20}$$

The matrices E and D are given by

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_t & x_{tr} & -x_{atd} & -x_{atq} & x_{atd} & x_{atq} \\ 0 & 0 & x_{tr} & x_r & -x_{ard} & x_{arq} & x_{ard} & -x_{arq} \\ 0 & 0 & x_{atd} & x_{ard} & -x_d & 0 & x_{ad} & 0 \\ 0 & 0 & x_{atq} & -x_{arq} & 0 & -x_q & 0 & x_{aq} \\ 0 & 0 & x_{atd} & x_{ard} & -x_{ad} & 0 & x_{kkd} & 0 \\ 0 & 0 & x_{atq} & -x_{arq} & 0 & -x_{aq} & 0 & x_{kkq} \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & w_o & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_o & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{M} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

The column vector f(X) is given as

$$f(X)=\begin{bmatrix} w-w_o \\ \left[-K_d(w-w_o)+(x_d-x_q)i_d i_q - x_{ad}i_{kd}i_q + x_{atq}i_d i_t + \right. \\ \left. x_{aq}i_d i_{kq} - x_{atd}i_q i_t - x_{ard}i_q i_r - x_{arq}i_d i_r\right]/M \\ -w_o\, r_t\, i_t \\ -w_o\, r_r\, i_r \\ w_o(v_d+r_a i_d)+w(x_{atq}i_t - x_{arq}i_r - x_q i_q + x_{aq}i_{kq}) \\ w_o(v_q+r_a i_q)+w(x_d i_d - x_{atd}i_t - x_{ard}i_r - x_{ad}i_{kd}) \\ -w_o\, r_{kd}\, i_{kd} \\ -w_o\, r_{kq}\, i_{kq} \end{bmatrix}$$

Linear State Model:

As discussed earlier when deriving a state space model for the normal synchronous machine, it will be convenient to linearize the system equations to implement the control theoretic concepts for the design of optimal regulators. It will be shown later that the control law obtained will be applicable even for large disturbances, i.e. when the system is described by its nonlinear equations. Therefore linearization of the system equation (7.18) about an operating point, leads to

$$E\,\dot{X} = F\,X + D\,u \qquad (7.21)$$

where X and u are the deviations of the state and control vectors about the operating point respectively. The operating point matrix F is given by

$$
F = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{K_d}{M} & f_{23} & f_{24} & f_{25} & f_{26} & -\frac{x_{ad} i_{do}}{M} & \frac{x_{aq} i_{qo}}{M} \\
0 & 0 & -w_o r_t & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -w_o r_r & 0 & 0 & 0 & 0 \\
f_{51} & f_{52} & w_o x_{atq} & -w_o x_{arq} & w_o(r_a+r_e) & f_{56} & 0 & w_o x_{aq} \\
f_{61} & f_{62} & -w_o x_{atd} & -w_o x_{ard} & f_{65} & w_o(r_a+r_e) & -w_o x_{ad} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -w_o r_{kd} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -w_o r_{kq}
\end{bmatrix}
$$

where

$$f_{23} = (x_{atq} i_{do} - x_{atd} i_{qo})/M$$

$$f_{24} = -(x_{ard} i_{qo} + x_{arq} i_{do})/M$$

$$f_{25} = [(x_d - x_q) i_{qo} + x_{atq} i_{to} + x_{aq} i_{kqo} - x_{arq} i_{ro}]/M$$

$$f_{26} = [(x_d - x_q) i_{do} - x_{ad} i_{kdo} - x_{atd} i_{to} - x_{ard} i_{ro}]/M$$

$$f_{51} = w_o V_o \cos \delta_o$$

$$f_{52} = x_{atq} i_{to} - x_{arq} i_{ro} - x_q i_{qo} + x_{aq} i_{kqo}$$

$$f_{56} = -w_o (x_e + x_q)$$

$$f_{61} = -w_o V_o \sin \delta_o$$

$$f_{62} = x_d i_{do} - x_{atd} i_{to} - x_{ard} i_{ro} - x_{ad} i_{kdo}$$

$$f_{65} = w_o (x_e + x_d)$$

Premultiplication of equation (7.21) by the inverse of E, gives the system state equation as

$$\dot{X} = A\ X + B\ u \tag{7.22}$$

The output variables are chosen as the rotor angle, rotor speed and the machine terminal voltage. By linearizing equation (7.15) gives

$$\Delta V_m = \frac{v_{do}}{V_{mo}} \Delta v_d + \frac{v_{qo}}{V_{mo}} \Delta v_q \tag{7.23}$$

Also linearising the equations (7.16) and (7.17),

$$\Delta v_d = V_o \operatorname{Cos} \delta_o \Delta\delta + r_e \Delta i_d - x_e \Delta i_q \tag{7.24}$$

$$\Delta v_q = -V_o \operatorname{Sin} \delta_o \Delta\delta + r_e \Delta i_q + x_e \Delta i_d \tag{7.25}$$

Therefore combining the above three equations,

$$\Delta v_m = c_1 \Delta\delta + c_5 \Delta i_d + c_6 \Delta i_q \tag{7.26}$$

where

$$c_1 = V_o(v_{do} \operatorname{Cos} \delta_o - v_{qo} \operatorname{Sin} \delta_o)/\bar{V}_{mo}$$

$$c_5 = (v_{do}\ r_e + v_{qo}\ x_e)/V_{mo}$$

$$c_6 = (v_{qo}\ r_e - v_{do}\ x_e)/V_{mo}$$

Thus the output equation for the system can be obtained as

$$Y = C\ X \tag{7.27}$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_1 & 0 & 0 & 0 & c_5 & c_6 & 0 & 0 \end{bmatrix}$$

and

$$Y = (\Delta\delta\ ,\ \Delta\ w,\ \Delta V_m)^T$$

Once the operating point is selected, the system matrices A, B and C can be immediately calculated. The complete system state space model is obtained as a linear time invariant model and is given by equations (7.22) and (7.27).

7.3 PERFORMANCE OF THE UNCONTROLLED SYSTEM

The system shown in Figure 7.3 has the following parameters, wherein all the quantities are in p.u. except time and angle which are in seconds and radians respectively[4].

$x_{ad} = 1.0$	$x_{aq} = 0.6$	$x_d = 1.2$	$x_q = 0.8$
$x_{kkd} = 1.1$	$x_{kkq} = 0.8$	$x_{al} = 0.2$	$x_{atd} = 0.87$
$x_{ard} = 0.87$	$x_{atq} = 0.5$	$x_{arq} = 0.3$	$x_{at} = 0.9$
$x_{ar} = 0.9$	$x_{tr} = 0.55$	$x_t = 1.1$	$x_r = 1.1$
$r_a = 0.01$	$r_{kd} = 0.02$	$r_{kq} = 0.04$	$r_t = 0.0011$
$r_r = 0.0011$	$r_e = 0.05$	$x_e = 0.3$	$V_o = 1.0$
$M = 0.0192$	$K_d = 0.0032$	$w_o = 314.0$	

At the chosen operating point the machine is delivering rated KVA at unity power factor to the infinite bus. For the above operating conditions the various machine quantities are calculated using the phasor diagram shown in Figure 7.4 and are given below :

$$\delta_o = 55.2° \quad i_{to} = 1.042 \quad i_{ro} = 1.213 \quad i_{do} = 0.82$$

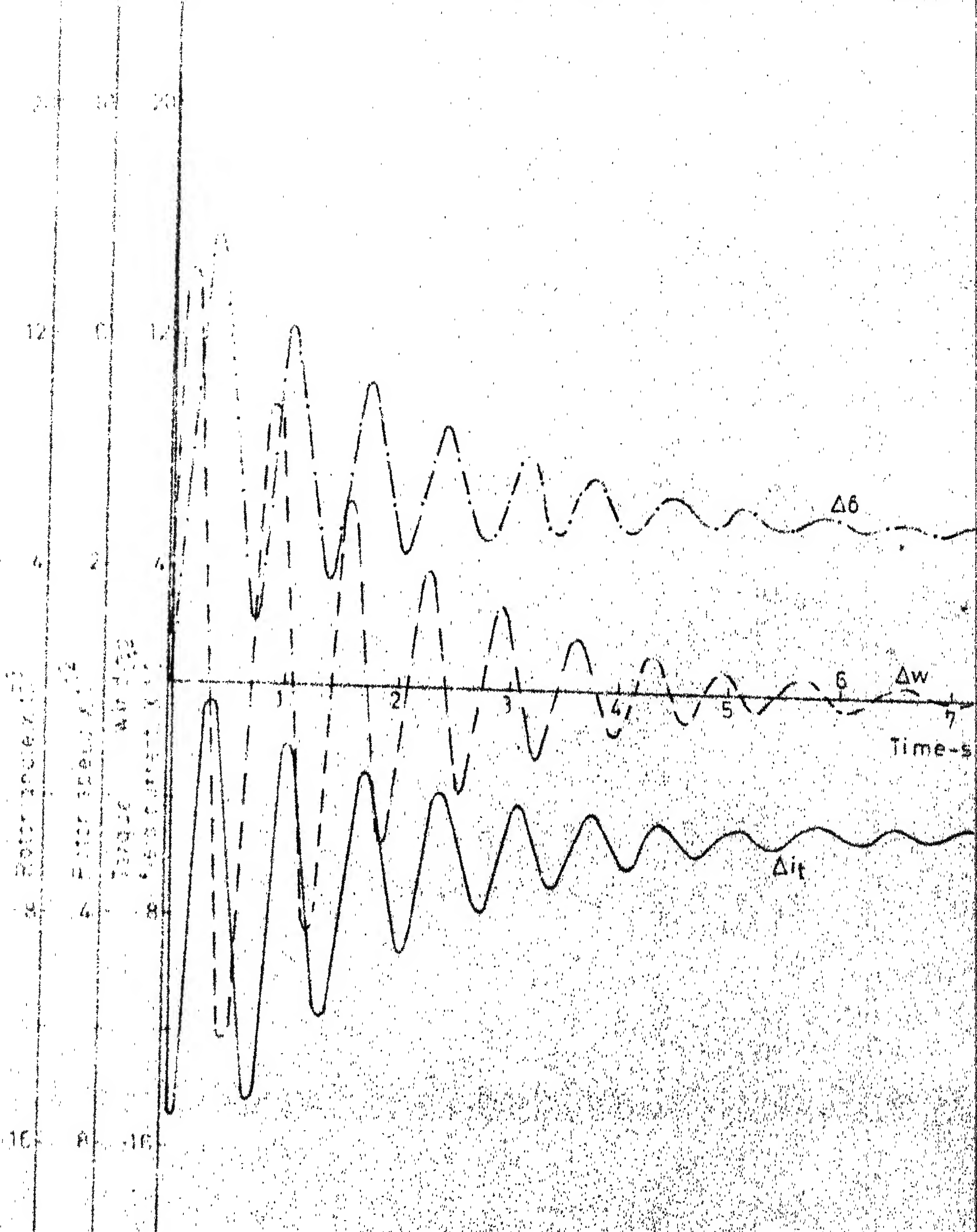


FIG. 7.5 D.W.R. MACHINE RESPONSE WITH NO REGULATORS

7.4 PERFORMANCE WITH CONVENTIONAL REGULATORS

In the conventional design procedure, suitable configurations for the regulators are chosen first and then the gains and time constants are adjusted to meet the design specifications and stability requirements. It is shown[31,32] that control of quadrature axis field winding from load angle feedback effectively extends the stable operation to cover the whole range of reactive power. For the system considered, a voltage regulator with one time constant and an angle regulator with single time constant are used for the excitation of reactive and torque windings respectively. A conventional speed governor with two time constants is used for the control of input torque. In the steady state these controllers will keep the terminal voltage and load angle at specified values irrespective of the loading conditions.

The performance equations for the regulators are given by[32] :

Voltage Regulator:

$$E_r = \frac{K_v}{1+T_v p}(V_{ref} - V_m) \tag{7.28}$$

Angle Regulator:

$$E_t = \frac{K_a}{1+T_a p} (\delta_{ref} + \delta) \tag{7.29}$$

Speed Governor[5]:

$$T_1 = \frac{K_g}{(1+T_g p)(1+T_h p)}(w_o - w)/w_o \tag{7.30}$$

where

$$E_t = v_t\, x_{atd}/r_t \quad \text{and} \quad E_r = v_r\, x_{ard}/r_r$$

Choosing the state variables for the controllers as ΔE_t, ΔE_r, ΔT_1 and $p\,\Delta T_1$, the equations (7.28) to (7.30) are thrown into state space form after linearizing, as

$$p\,\Delta E_t = \frac{K_a}{T_a}\Delta\delta - \frac{1}{T_a}\Delta E_t \tag{7.31}$$

$$p\,\Delta E_r = -\frac{K_v}{T_v} c_1 \Delta\delta - \frac{K_v}{T_v} c_5 \Delta i_d - \frac{K_v}{T_v} c_6 \Delta i_q - \frac{1}{T_v}\Delta E_r \quad \ldots \tag{7.32}$$

$$p\,\Delta T_1 = \Delta T_2 \tag{7.33}$$

$$p\,\Delta T_2 = -\frac{K_g}{w_o T_g T_h}\Delta w - \frac{1}{T_g T_h}\Delta T_1 - \frac{T_g + T_h}{T_g T_h}\Delta T_2 \tag{7.34}$$

Combining these equations with system state equation (7.22), the controlled system state model is obtained as

$$\dot{X} = A_1 X \tag{7.35}$$

The various regulator parameters are selected as[34]

$K_a = 2.0$ $\quad K_v = 5.0$ $\quad K_g = 5.0$

$T_a = 0.5$ $\quad T_v = 0.25$ $\quad T_g = 0.1$

$T_h = 0.5$

The system matrix A_1 for the operating point considered in the earlier section is given by

$$
A_1 = \begin{bmatrix}
0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & -0.17 & -13.02 & -38.64 & 9.20 & -85.09 & -29.69 & 25.53 & 0.00 & 0.00 & 52.08 & 0.00 \\
-24.45 & 1.31 & 87.59 & 310.69 & -366.83 & 393.06 & 244.55 & -212.73 & 1.54 & 0.18 & 0.00 & 0.00 \\
-399.76 & -0.14 & -511.01 & -90.27 & 322.44 & 420.63 & -214.96 & -231.11 & 0.18 & 1.54 & 0.00 & 0.00 \\
-730.85 & 2.07 & -384.55 & 387.74 & -76.83 & 1408.54 & -11.58 & -750.29 & 0.47 & 0.47 & 0.00 & 0.00 \\
817.51 & 3.10 & 860.93 & 866.43 & -1493.41 & -59.75 & 995.61 & -22.97 & 0.29 & -0.29 & 0.00 & 0.00 \\
-330.49 & 0.94 & -172.89 & 174.52 & -34.74 & 636.93 & -39.64 & -347.42 & -0.94 & -0.94 & 0.00 & 0.00 \\
471.64 & 1.79 & 499.87 & 499.47 & -861.58 & -34.45 & 574.39 & -39.82 & -0.29 & 0.29 & 0.00 & 0.00 \\
4.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -2.00 & 0.00 & 0.00 & 0.00 \\
5.49 & 0.00 & 0.00 & 0.00 & -5.28 & 3.05 & 0.00 & 0.00 & 0.00 & -4.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\
0.00 & -0.32 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -20.00 & 12.00
\end{bmatrix}
$$

The response of the controlled system for an initial disturbance in the torque winding field current is obtained by solving the system equation (7.35). The response is shown in Figure 7.6. The oscillatory response dies down in about six seconds. Even though the response is better than the response shown in Figure 7.5, it is still oscillatory. An improper choice of regulator parameters can make the system unstable.

7.5 CONCLUSION

The need for the use of an additional field winding to improve the dynamic performance of the synchronous machines is briefly discussed. For d.w.r. synchronous machine, the state space model is derived in a linear time invariant form. The machine is considered with simple angle and voltage regulators on the field windings and a speed governor for controlling the speed. The performance of the system with and without conventional regulators is obtained using the state space model. The responses are oscillatory and take longer times to settle down. The feasibility of using either optimal or suboptimal constant feedback control law for the d.w.r. machine is considered in the next chapter, to improve the system dynamic performance.

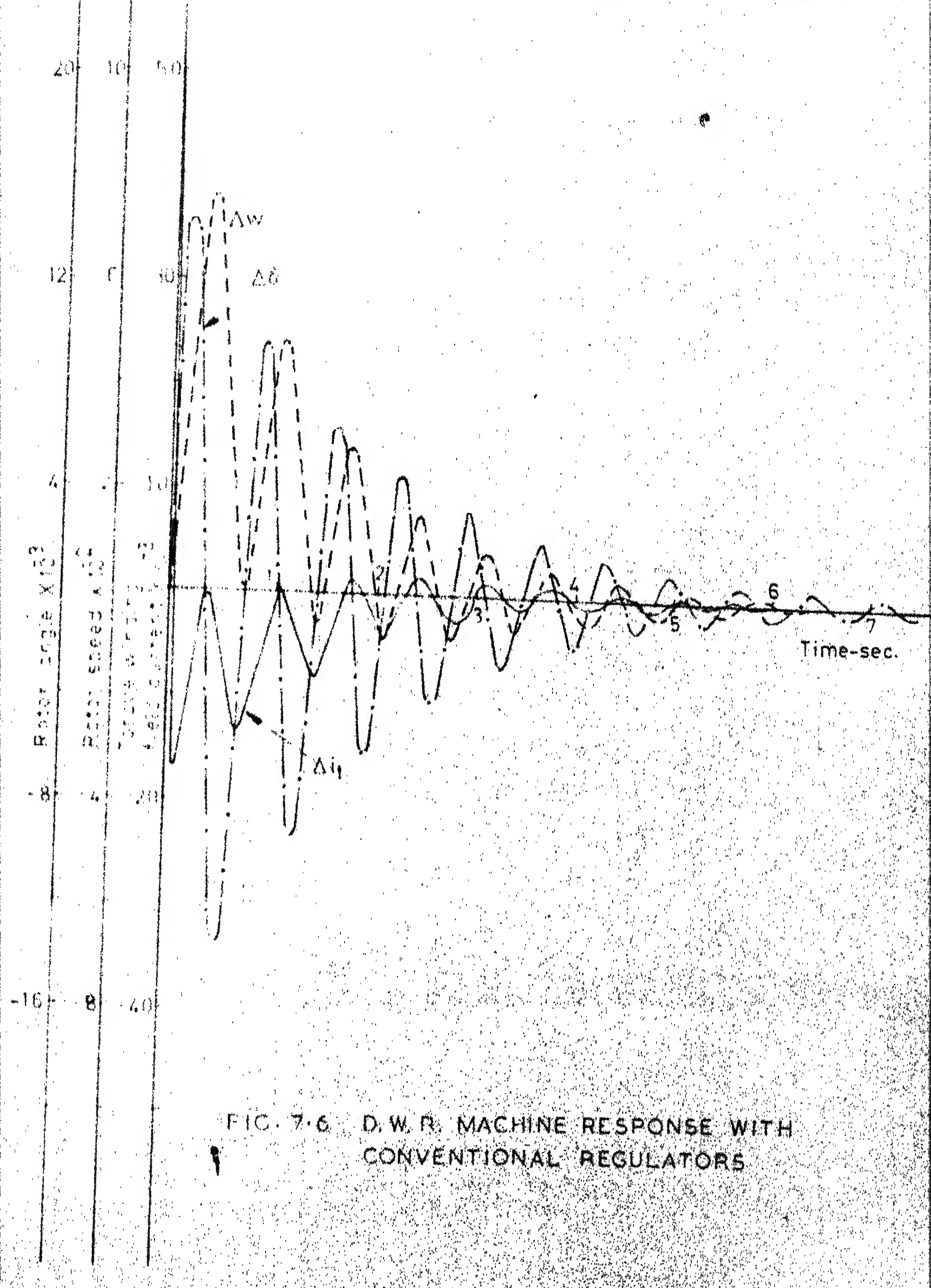


FIG. 7·6 D.W.R. MACHINE RESPONSE WITH CONVENTIONAL REGULATORS

CHAPTER VIII

OPTIMAL AND SUBOPTIMAL CONTROL OF D.W.R.SYNCHRONOUS GENERATOR

8.1 INTRODUCTION

An optimal state regulator is obtained for the d.w.r. synchronous machine using the state space model derived in the previous chapter. The optimal control law requires the availability of the entire state variables for measurements. But in many practical problems this is difficult to achieve for physical reasons. Hence a dynamic observer is discussed which reconstructs the unavailable state variables from the available output measurements. The observer introduces exponentially decaying error in the reconstructed state variables and also transfer functions are introduced in the feedback paths. Also the estimation of state variables becomes a difficult task where there are a large number of state variables but only fewer output variables. In such cases, it is most desirable to obtain an optimal regulator which is a function of the measurable outputs. If this relationship is linear and time invariant, then the control can be easily implemented.

A linear constant output feedback control of the d.w.r. synchronous machine is also discussed. The

performance of the system with optimal and suboptimal regulators and with dynamic observer is obtained for impulse type disturbances. The responses are compared and conclusions are drawn.

8.2 OPTIMAL REGULATOR FOR D.W.R. MACHINE

The classical methods of control systems design for synchronous machines assume apriori configuration of the regulators. The feedback signal is assumed to be derived from some output quantity; for example, a signal proportional to the terminal voltage is fed to the field winding to maintain the terminal voltage at desired levels. In modern control systems practice, an integrated form of control is derived which is a function of the states or outputs. Thus an optimal or a suboptimal regulator is obtained by a proper choice of a performance criterion.

A quadratic performance index in the state and control variables is selected for the design of an optimal regulator for the d.w.r. machine. The problem is therefore to find a control law which minimizes the performance index

$$J = \frac{1}{2} \int_0^{\infty} (X^T Q X + u^T R u) \, dt \tag{8.1}$$

subject to the state equation

$$\dot{X} = A X + B u \tag{8.2}$$

The state model given by equation (8.2) is derived in Chapter VII. The optimal control law is shown to be[17]

$$u = - R^{-1} B^{T} P X \qquad (8.3)$$

where P is the positive definite solution of

$$P A + A^{T} P - P B R^{-1} B^{T} P + Q = 0 \qquad (8.4)$$

and the optimally controlled system is given by

$$\dot{X} = (A - B R^{-1} B^{T} P) X \qquad (8.5)$$

The operating point considered is that the d.w.r. machine is delivering rated KVA at rated voltage at unity power factor to the infinite bus. For this load condition the system matrices A and B are given in the previous chapter. The weightage matrices for the state and control variables are chosen as discussed in Chapter IV and are given by Q = dia (1,1,1,1,1,1,1,1) and R = dia (1,1,1). With these matrices, the Riccati equation (8.4) is solved by the method of successive approximation (discussed in Chapter IV) and the P-matrix is obtained as

$$P = \begin{bmatrix} 1.5227 & 0.0006 & 0.2590 & 0.2034 & -0.2733 & -0.0614 & 0.2873 & 0.0699 \\ 0.0006 & 0.0188 & -0.0172 & -0.0129 & 0.0184 & 0.0037 & -0.0184 & -0.0060 \\ 0.2590 & -0.0172 & 0.2807 & 0.2430 & -0.3100 & -0.0399 & 0.3249 & 0.0477 \\ 0.2034 & -0.0129 & 0.2430 & 0.2616 & -0.2988 & 0.0208 & 0.3136 & -0.0188 \\ -0.2733 & 0.0184 & -0.3100 & -0.2988 & 0.3645 & 0.0114 & -0.3769 & -0.0174 \\ -0.0614 & 0.0037 & -0.0399 & 0.0208 & 0.0114 & 0.0708 & -0.0108 & -0.0718 \\ 0.2873 & -0.0184 & 0.3249 & 0.3136 & -0.3769 & -0.0108 & 0.3981 & 0.0172 \\ 0.0699 & -0.0060 & 0.0477 & -0.0188 & -0.0174 & -0.0718 & 0.0172 & 0.0836 \end{bmatrix}$$

The optimally controlled system given by equation (8.5) is then solved for the dynamic performance when there is an initial disturbance of 0.05 p.u. in the torque winding field current. The optimal regulator response is shown in Figure 8.1. The response is nonoscillatory and decays exponentially very fast. By comparison of this response with conventional regulator response shown in Figure 7.5, it can be concluded that the optimal regulators are superior in performance than the conventional regulators.

The optimal control law is obtained at different operating conditions and the average values of the performance index for these conditions are calculated using equation (6.20). The machine is delivering rated KVA at rated voltage to the infinite bus, at the different power factors. The performance index values are given below for the different conditions.

Power factor	0.4 lag	0.8 lag	Unity	0.8 lead	0.4 lead
$\dot{J}$	3.524	3.318	3.008	2.863	2.849

From the above results it can be concluded that the optimal regulator obtained at one operating condition can be used at different load conditions without much performance deterioration.

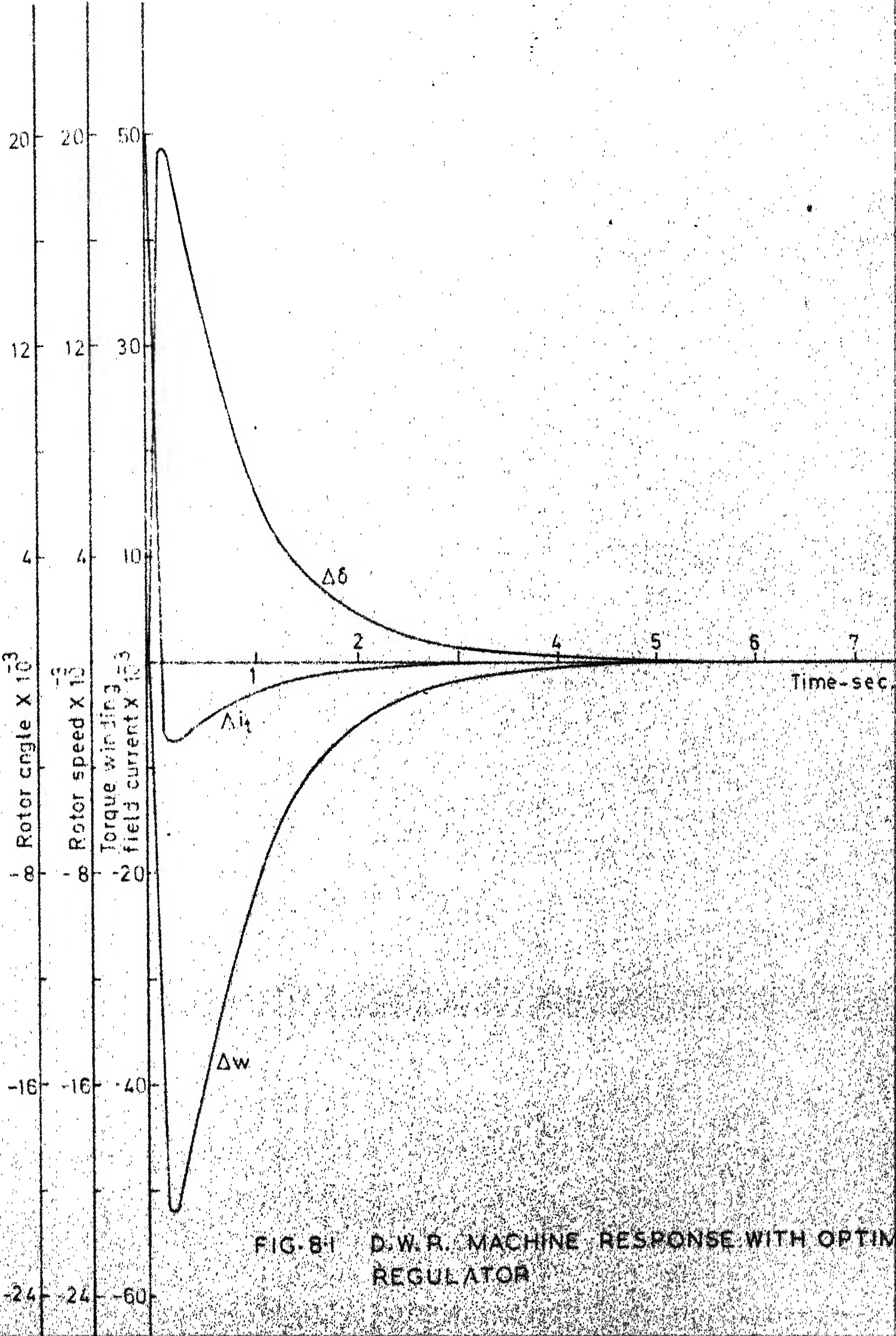


FIG. 8·1 D.W.R. MACHINE RESPONSE WITH OPTIM
REGULATOR

8.3 PERFORMANCE OF THE OPTIMAL REGULATOR FOR LINE RECLOSURE

The system shown in Figure 7.3 is operating initially with one line in service and has the following operating conditions in steady state :

$$\delta_o = 65.8^\circ \quad w_o = 314.0 \quad i_{to} = 0.934 \quad i_{ro} = 1.638$$

$$i_{do} = 0.91 \quad i_{qo} = 0.412 \quad v_{do} = 0.76 \quad v_{qo} = 1.0$$

The problem is to investigate the performance of the system with optimal regulators, when the second line is reclosed. With two lines in service the operating conditions are already given in the previous chapter. From the control theory point of view, the problem is to transfer the system state from one line service conditions to two lines operating conditions. The performance of the d.w.r. machine for this disturbance with conventional regulators, discussed in the previous chapter, is shown in Figure 8.2 for this system state transfer. With optimal regulators the performance is shown in Figure 8.3. By comparing these two responses, it can be established that the optimal regulator is better than conventional regulators even for such large disturbances.

8.4 PERFORMANCE OF D.W.R. MACHINE WITH DYNAMIC OBSERVER

The optimal regulator discussed in the last section calls for the measurement of entire state vector for feedback. But it is seldom that all the state variables

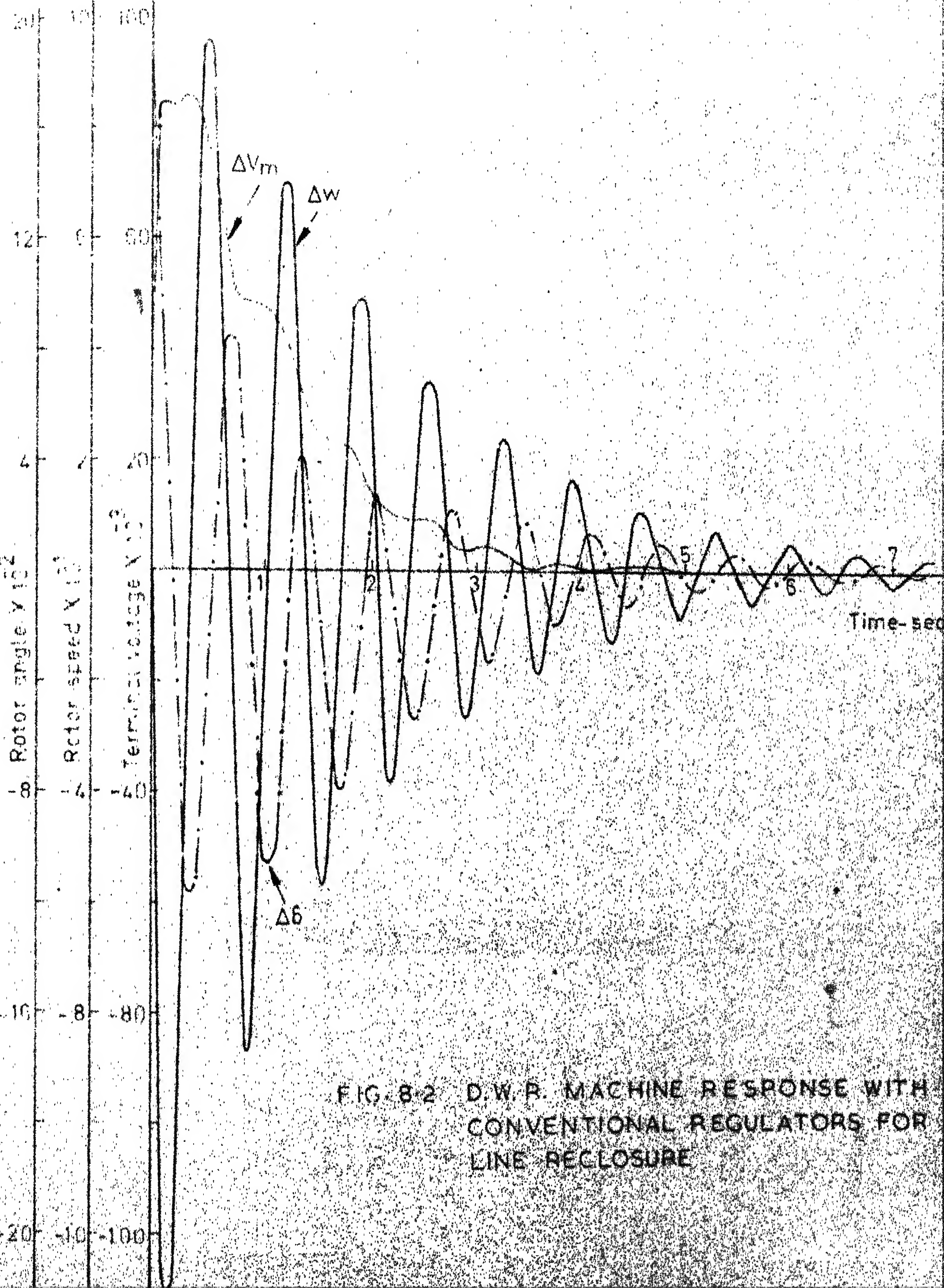


FIG. 8.2 D.W.R. MACHINE RESPONSE WITH CONVENTIONAL REGULATORS FOR LINE RECLOSURE

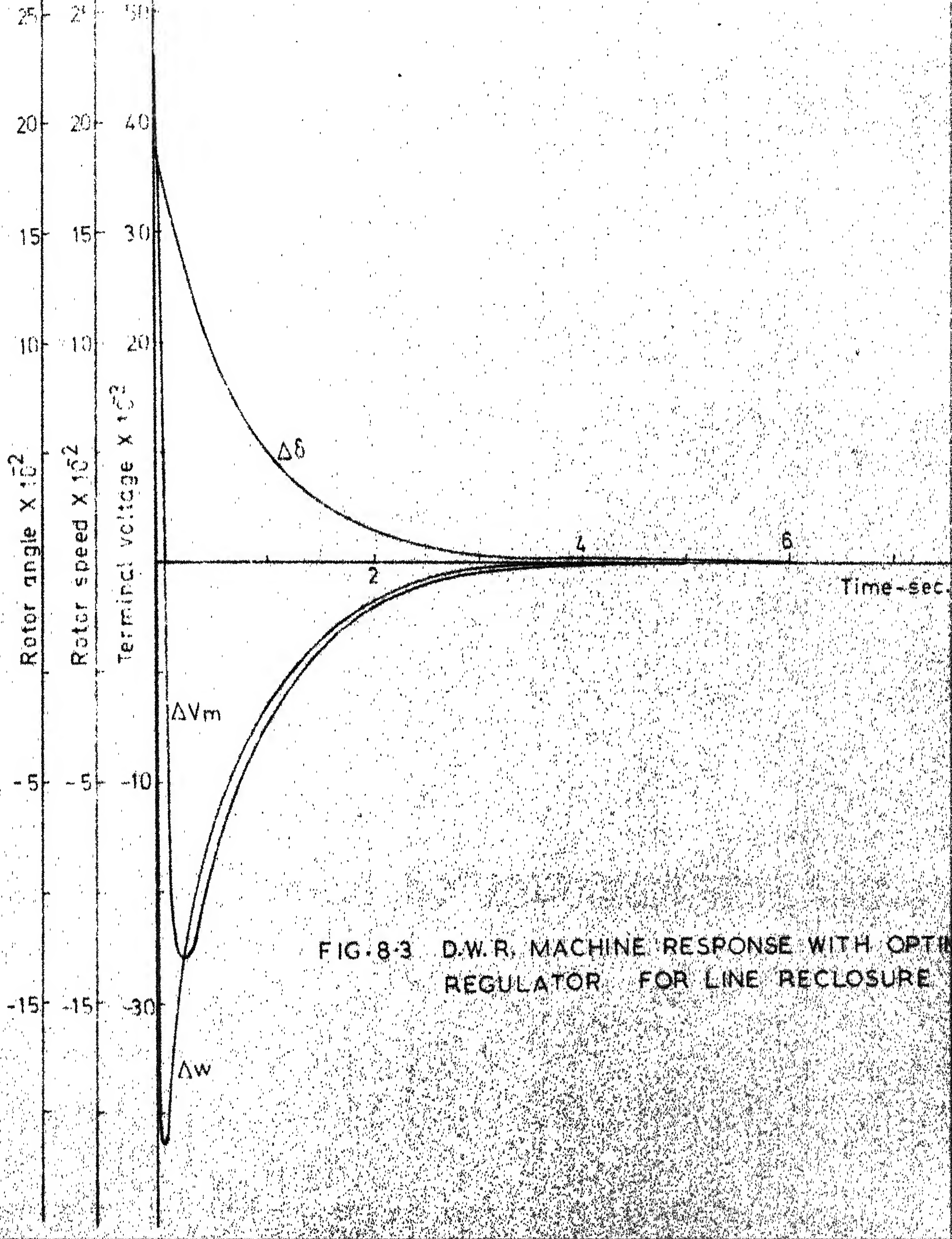


FIG.8.3 D.W.R. MACHINE RESPONSE WITH OPTI
REGULATOR FOR LINE RECLOSURE

can be measured directly. In this section, a compatible dynamic observer is obtained which reconstructs the complete state vector from the output measurements. The output equation for the system under consideration is derived in the previous chapter and is given by

$$Y = C X \tag{8.6}$$

The dynamic observer is discussed in detail in Chapter V. For the d.w.r. machine, the eight state variables must be reconstructed from the three output variables namely $\Delta\delta$, Δw and ΔV_m. Thus the order of the compatible dynamic observer becomes five. The observer matrices F and G are selected such that F is a stable matrix and the pair (F,G) is controllable and are given by

$$F = \begin{bmatrix} -10.0 & 0 & 0 & 0 & 0 \\ 0 & -10.0 & 0 & 0 & 0 \\ 0 & 0 & -10.0 & 0 & 0 \\ 0 & 0 & 0 & -10.0 & 0 \\ 0 & 0 & 0 & 0 & -10.0 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Given the matrices F and G as above and the system matrices A, B and C in Chapter VII for the specified operating conditions, the observer matrices T, H and W are calculated following the procedure discussed in Chapter V. These are given below :

$$T = \begin{bmatrix} 0.0536 & -0.0055 & -0.0225 & -0.0257 & 0.0351 & 0.0074 & -0.0349 & -0.0058 \\ 0.4095 & 0.0544 & 0.3992 & 0.2261 & -0.3354 & -0.0743 & 0.0310 & -0.0193 \\ 0.3370 & -0.0003 & -0.0460 & -0.0738 & 0.0155 & 0.1192 & 0.0157 & -0.0028 \\ 0.0873 & -0.0057 & -0.0885 & -0.0995 & 0.0506 & -0.1182 & -0.0192 & -0.0086 \\ 0.4432 & 0.0541 & 0.3462 & 0.1523 & -0.3198 & -0.1935 & 0.3259 & -0.2214 \end{bmatrix}$$

$$H = \begin{bmatrix} -13.5700 & 143.30 & -99.150 & -112.70 & 44.17 \\ --1.6380 & -9.41 & -75.460 & -77.10 & -84.87 \\ -0.2839 & 2.83 & -0.014 & -29.79 & 2.82 \end{bmatrix}^T$$

$$W = \begin{bmatrix} 1.000 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.2744 & 0.0 & 0.0 & 0.0 & 0.2641 & -0.1514 & 0.0 & 0.0 \\ 0.0536 & -0.0055 & -0.0425 & -0.0257 & 0.0351 & 0.0074 & -0.0349 & -0.006 \\ 0.4095 & 0.0544 & 0.3992 & 0.2261 & -0.3354 & -0.0743 & 0.0310 & -0.019 \\ 0.3370 & -0.0003 & -0.0460 & -0.0738 & 0.0155 & 0.1192 & 0.0157 & -0.003 \\ 0.0873 & -0.0057 & -0.0885 & -0.0995 & 0.0506 & -0.1182 & -0.0192 & -0.008 \\ 0.4432 & 0.0541 & 0.3462 & 0.1533 & -0.3198 & -0.1935 & 0.3259 & -0.221 \end{bmatrix}$$

Once these observer matrices are computed, the observer dynamics are completely specified. Then the observer is cascaded with the optimal regulator to obtain

the overall control system. The performance of the cascaded system for an initial disturbance of 0.05 p.u. in the torque winding field current is shown in Figure-8.4. The responses of the variables shown have large overshoot in the initial portions but decay fast exponentially. By choosing large negative eigen values for the matrix F, the error in the estimated state variables can be reduced. But this introduces large gains in the feedback paths and a compromise is essential between these two. It is also noted that the observer dynamics are very much sensitive to the operating conditions and hence the overall control will be different for different load conditions. Thus this type of optimal control will have limited applications.

8.5 SUBOPTIMAL CONTROL OF D.W.R. MACHINE

The optimal regulator gives the best performance for a chosen performance criterion. Unfortunately, it cannot be implemented in many physical problems. The question might, therefore, be asked whether a suboptimal control law can be used which is easy to implement. In this section, a control law is obtained which is a function of the output variables only. The control law is thus constrained to be

$$u = F\,Y \tag{8.7}$$

The problem is thus modified which requires the solution of the feedback control matrix F that minimizes the

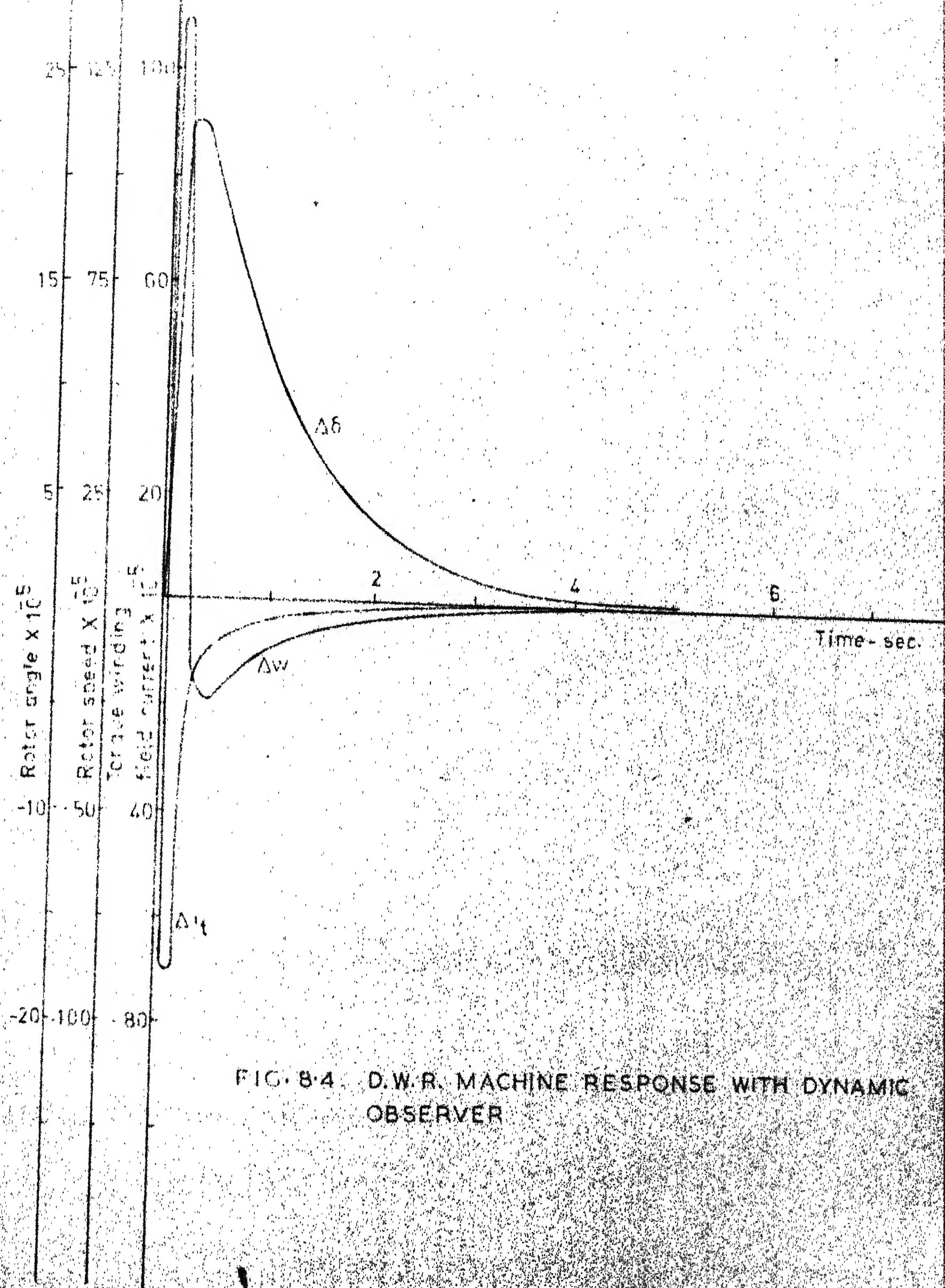


FIG. 8·4 D.W.R. MACHINE RESPONSE WITH DYNAMIC OBSERVER

performance index given by equation (8.1) subject to the state and output equations (8.2) and (8.6) respectively.

The optimal feedback matrix is shown[26] to be the solution of equations (6.15) to (6.17) as discussed in Chapter VI. Using these equations and the system matrices A, B and C for the operating point considered in Chapter VII, the optimal feedback matrix is obtained by the iterative algorithm[26] as

$$F = \begin{bmatrix} 0.001175 & 0.00218 & -0.002318 \\ -0.001728 & -0.001338 & -0.004625 \\ -0.5941 & -1,4367 & 1.322 \end{bmatrix}$$

With this feedback suboptimal control matrix F, the controlled system is solved for the response when there is an *initial* disturbance in the torque winding field current and the response is shown in Figure 8.5. The comparison of this response with the optimal response (Figure 8.1) shows that the suboptimal control is good enough and it takes longer time than the optimal one for the responses to settle down and it is also nonoscillatory. The implementation of the control law is simple because it requires only the output variables for feedback. The performance deviation is not very much as can be seen by comparison of the two responses.

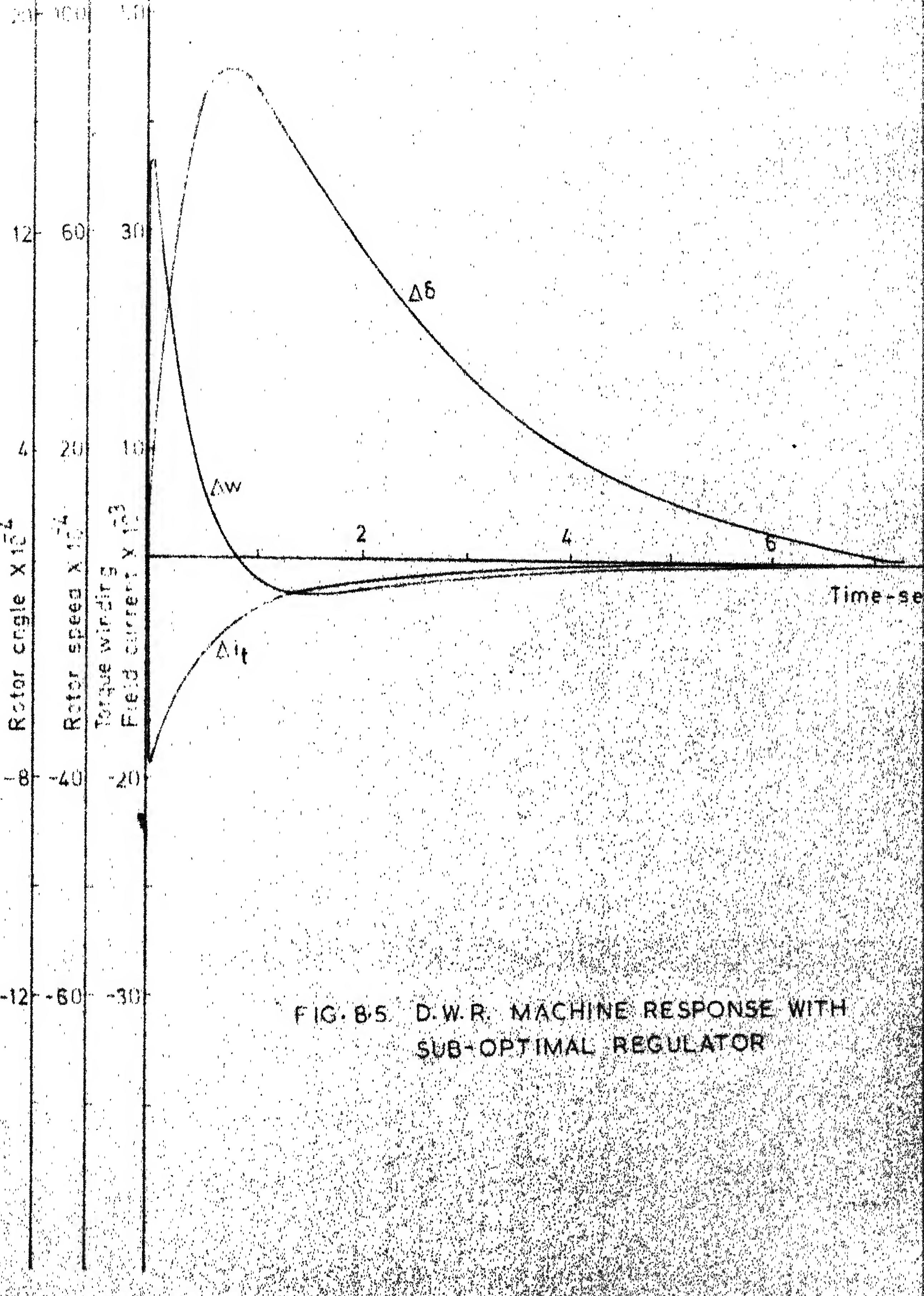


FIG. 8.5 D.W.R. MACHINE RESPONSE WITH SUB-OPTIMAL REGULATOR

The suboptimal control law obtained for the above operating conditions is used at different loads. The average performance values by use of the suboptimal control calculated at unity power factor and at rated KVA, when used at different operating conditions are given below. The machine is delivering rated KVA at different power factors to the infinite bus.

Power Factor	0.4 lag	0.8 lag	Unity	0.8 lead
$\hat{J}$	18.025	17.0024	16.96	17.624

From the above results, it can be inferred that the use of a particular control law obtained at one operating point can be used over wide range of load conditions with less performance deterioration. The performance of the system with this suboptimal control law is investigated for large disturbance such as line reclosure as discussed in Section 8.3. The performance obtained with suboptimal control is shown in Figure 8.6. Comparison of this response with the conventional regulator response (Figure 8.2) for the same disturbance reveals that suboptimal regulators are better than the conventional ones.

As discussed in Chapter VI, the measurement of the output variables except the rotor angle is easy and

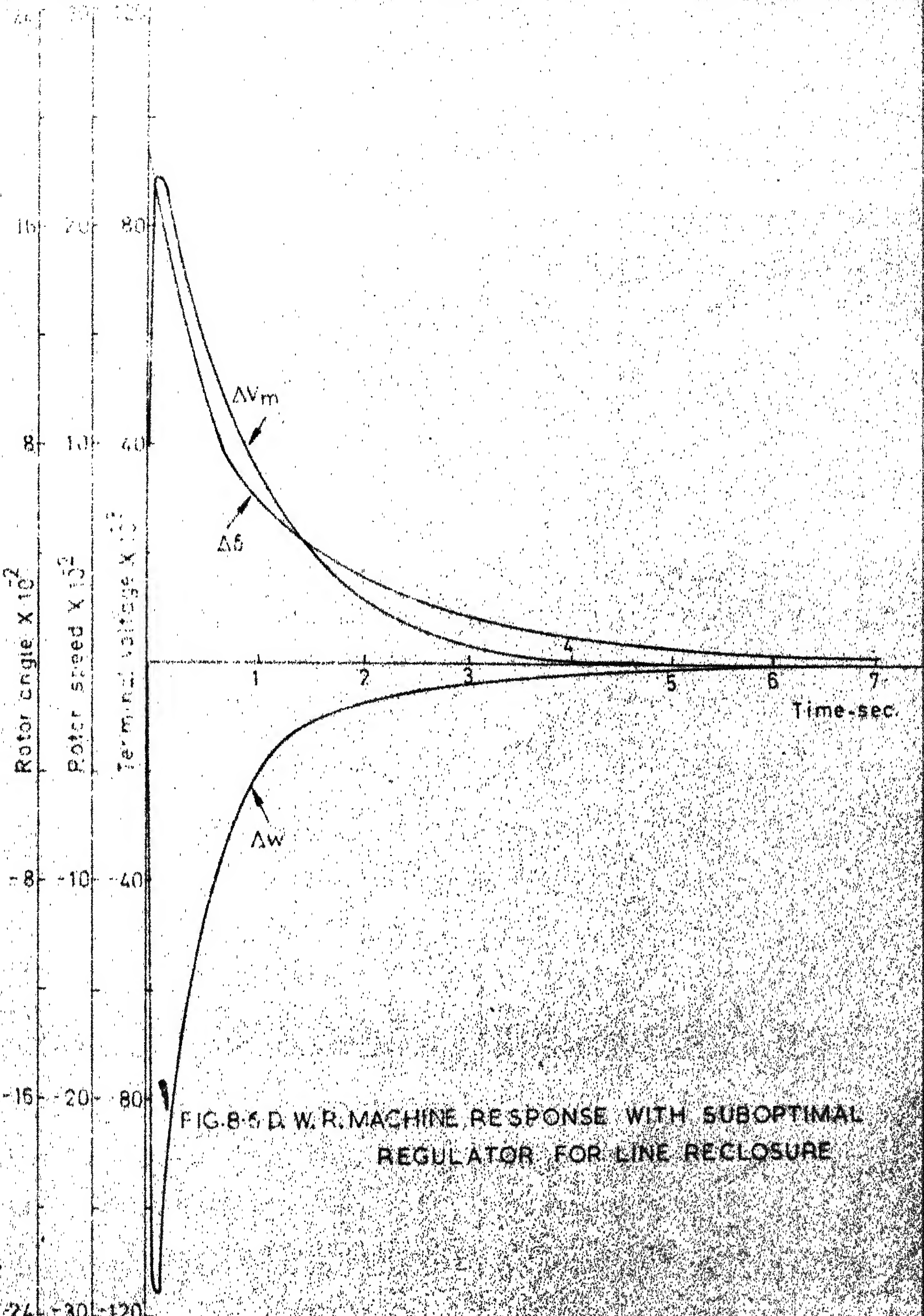


FIG.8.5 D.W.R. MACHINE RESPONSE WITH SUBOPTIMAL REGULATOR FOR LINE RECLOSURE

also their steady state values are known apriori. But the system rotor angle with reference to the infinite bus changes with the load conditions. Instead of measuring δ with respect to the infinite bus, the angle between terminal voltage and the rotor quadrature axis can be measured[31]. In the case of d.w.r. machine, this angle is maintained constant at 30°E by proper excitation of the torque and reactive windings for all operating conditions. Thus the rotor angle feedback can be obtained from the measurement of this angle[31]. Then the steady state values of ΔV_m, $\Delta\delta$ and Δw are zero and their reference values are specified. Therefore this suboptimal control can be easily implemented on practical systems.

8.6 CONCLUSION

An optimal state regulator is obtained for the d.w.r. synchronous machine using the state space model derived in the previous chapter. The optimality of the regulator at different load conditions is then investigated. Since the implementation of the optimal control law requires the availability of the entire state variables for direct monitoring, it is difficult in most cases to implement a truly optimal control. Thus the reconstruction of the unavailable state variables by state reconstruction is considered. The response of the optimal system with dynamic observer is obtained and

compared with the optimal response. The observer dynamics are very much sensitive to the operating conditions and hence they are different for different load levels unlike the optimal control law. Also the observer introduces error in the state variables. To obviate these difficulties, a suboptimal control is desired which is easy to implement. The suboptimal control law is obtained using the trace minimization procedure[26]. The performance of the system with suboptimal regulator is then determined. By comparison of the responses, it can be concluded that the suboptimal control is most suitable both from the point of implementation and system dynamic performance.

In all the above treatment, the problem considered is a deterministic one. But in practical problems there are always random disturbances and measurement errors. Therefore, any design should take into account the presence of these random noises. In the next chapter, the design of optimal regulators in the presence of such stochastic disturbances is discussed.

CHAPTER IX

STOCHASTIC OPTIMAL CONTROL OF SYNCHRONOUS MACHINE

9.1 INTRODUCTION

The optimal control of synchronous machine in the presence of random disturbances in the state and uncertainty in the output measurements is discussed in this chapter. The increase in the value of the average performance index is then investigated. The average behaviour of the system in the presence of the white noise is then determined.

The superiority of optimal regulators over conventional regulators for the synchronous machine is well established in the previous chapters. The optimal or suboptimal regulators required the measurement of the state or output, for feedback and it is assumed that these variables can be measured exactly. However practical measuring instruments frequently give rise to some errors. Also the disturbances coming on the system are random in nature. Hence it is necessary to consider the effect of these random disturbances when designing optimal or suboptimal regulators.

9.2 STATEMENT OF STOCHASTIC PROBLEM

The stochastic problem discussed here is that it is required to find an optimal control u as a result of minimization of expected value of J given by

$$\hat{J} = E\left[\lim_{t_f \to \infty} \frac{1}{2t_f} \int_0^{t_f} (X^T Q_1 X + u^T R_1 u)\, dt\right] \quad (9.1)$$

for the linear time invariant dynamic system

$$\dot{X} = A X + B u + w \quad (9.2)$$

$$Y = C X + v \quad (9.3)$$

where w and v are zero mean valued Gaussian white noise terms in the system state and output measurements respectively. The covariance of those noise terms are defined as

$$E(w\, w^T) = Q_2\, \delta(t-\tau) \quad (9.4)$$

$$E(v\, v^T) = R_2\, \delta(t-\tau) \quad (9.5)$$

$$E(v\, w^T) = 0 \quad (9.6)$$

Q_1 and R_1 are the weightage matrices on the state and control variables respectively. Also it is assumed that Q_1 and Q_2 are positive semidefinite matrices and R_1 and R_2 are positive definite matrices. The noise terms are independent of the initial state $X(o)$ and the covariance of the initial state is given as

$$E[X(o)\, x(o)^T] = P_o \quad (9.7)$$

Using the separation theorem[24], optimal control law is derived by minimizing the performance index given by equation (9.1). The optimal control law is shown to be the same as that for the deterministic system, for the type of disturbances considered in this chapter. The optimal control law is thus given by[35]

$$u = - F \hat{X} \tag{9.8}$$

The controlled system and estimator states are coupled. The controlled system is given by

$$\dot{X} = A X - B F \hat{X} + w \tag{9.9}$$

and the state estimator is

$$\dot{\hat{X}} = A \hat{X} - B F \hat{X} + S(Y - C \hat{X}) \tag{9.10}$$

where

$$F = R_1^{-1} B^T P \tag{9.11}$$

$$S = K C^T R_2^{-1} \tag{9.12}$$

P and K are the solutions of Riccati equations

$$P A + A^T P - F^T R_1 F + Q_1 = 0 \tag{9.13}$$

$$A K + K A^T - S R_2 S^T + Q_2 = 0 \tag{9.14}$$

and $\hat{X}$ is the maximum likelyhood estimate of the state vector X. The mean square histories of the state X and estimated state $\hat{X}$ and their cross correlations are defined as

$$x = E(X X^T) \tag{9.15}$$

and

$$\hat{X} = E(\hat{X}\,\hat{X}^T) \tag{9.16}$$

The solution of the following differential equation gives the covariance matrix $\hat{X}$ as[35]

$$\dot{\hat{X}} = (A - B\,F)\hat{X} + \hat{X}(A - B\,F)^T + S\,R_2\,S^T \tag{9.17}$$

Then the covariance of X is obtained as

$$X = \hat{X} + K \tag{9.18}$$

For steady state average behaviour of the system, the solution of equation (9.17) converges to a constant value. Thus the steady state covariance matrix for the estimated state vector $\hat{X}$ is obtained by solving the following equations; when the matrices A, B, F, R_2, K etc. are constant :

$$(A - B\,F)\hat{X} + \hat{X}(A - B\,F)^T + S\,R_2\,S^T = 0 \tag{9.19}$$

The increase in the value of the performance index due to the presence of noise is shown to be[35]

$$\Delta J = \frac{1}{2}\,\mathrm{tr}(P\,Q_2 + F^T\,R_2\,F\,K) \tag{9.20}$$

From equation (9.20) it is seen that the presence of noise ($K \neq 0$, $Q_2 \neq 0$) increases the performance index on an average.

9.3 SYNCHRONOUS MACHINE SYSTEM

The system considered in Chapter II is investigated for the system dynamic behaviour in the presence

of noise in the state and/or control and in the output measurements. The system matrices A, B and C are given in Chapter II for the operating conditions at $\delta = 26.3°$. The noise present in the state and output are assumed to be white noise with zero mean values and with constant covariances. The weightage matrices on the state vector X and on the control vector u are chosen as $Q_1 = \text{dia}(1,1,1,1,1,1,1)$ and $R_1 = \text{dia}(1,1)$ respectively.

The covariance matrices Q_2 and R_2 of the noise terms w and v are selected for illustration as $Q_2 = \text{dia}(0.05, 0.05, 0.05, 0.05, 0.05, 0.05, 0.05)$ and $R_2 = \text{dia}(0.25, 0.25, 0.25)$ respectively. With these system matrices the Riccati equation (9.12) is solved by the method of successive approximation and the value of P-matrix is obtained as

$$P = \begin{bmatrix} 2.3747 & 0.0038 & 1.0652 & -0.9802 & 0.9797 & -0.1093 & 0.1198 \\ 0.0038 & 0.0192 & -0.0036 & 0.0055 & -0.0047 & 0.0051 & -0.0079 \\ 1.0652 & -0.0036 & 4.4356 & -4.0986 & 4.0892 & -0.0092 & 0.0148 \\ -0.9802 & 0.0055 & -4.0986 & 3.7608 & -3.7503 & 0.0104 & 0.0159 \\ 0.9797 & -0.0047 & 4.0892 & -3.7503 & 3.7538 & -0.0107 & 0.0161 \\ -0.1093 & 0.0051 & -0.0092 & 0.0104 & -0.0107 & 0.0844 & -0.0811 \\ 0.1198 & -0.0079 & 0.0148 & 0.0159 & 0.0161 & -0.0811 & 0.0911 \end{bmatrix}$$

Then the equation (9.13) is solved using the same method for the steady state solution of K-matrix and is given by

$$K = \begin{bmatrix} 0.0573 & -0.0150 & -0.1509 & -0.0903 & -0.0074 & 0.0411 & -0.0010 \\ -0.0150 & 1.1523 & -0.1041 & -0.0746 & -0.0034 & -0.0038 & 0.0262 \\ -0.1509 & -0.1041 & 0.5701 & 0.3639 & 0.0367 & -0.0960 & 0.0116 \\ -0.0903 & -0.0746 & 0.3639 & 0.2377 & 0.0276 & -0.0556 & 0.0080 \\ -0.0074 & -0.0034 & 0.0367 & 0.0276 & 0.0067 & -0.0035 & 0.0015 \\ 0.0411 & -0.0028 & -0.0960 & -0.0556 & -0.0035 & 0.0346 & 0.0051 \\ -0.0010 & 0.0262 & 0.0116 & 0.0080 & 0.0015 & 0.0051 & 0.0080 \end{bmatrix}$$

Once the matrices P and K are obtained, the matrices F and S can be determined using equation (9.10) and (9.11) respectively. Then the controlled system given by equation (9.9) is completely specified. The mean square histories of the estimated state $\hat{X}$ and their cross correlations are calculated using the matrices F and K. Then the steady state covariance matrix X is obtained from equation (9.18) as

$$X = \begin{bmatrix} 0.0725 & -0.0250 & -0.1974 & -0.1196 & -0.0111 & 0.4510 & -0.0025 \\ -0.0250 & 1.2140 & -0.0850 & -0.0633 & -0.0015 & -0.0084 & 0.0295 \\ -0.1974 & -0.0850 & 0.7174 & 0.4571 & 0.0481 & -0.1266 & 0.0155 \\ -0.1196 & -0.0633 & 0.4571 & 0.2967 & 0.0347 & -0.0749 & 0.0105 \\ -0.0111 & -0.0015 & 0.0481 & 0.0347 & 0.0076 & -0.0058 & 0.0018 \\ 0.0510 & -0.0084 & -0.1266 & -0.0749 & -0.0058 & 0.0411 & 0.0042 \\ -0.0025 & 0.0295 & 0.0155 & 0.0105 & 0.0018 & 0.0042 & 0.0083 \end{bmatrix}$$

The increase in the value of performance index is calculated using equation (9.19) as $\Delta J = 1.03122$.

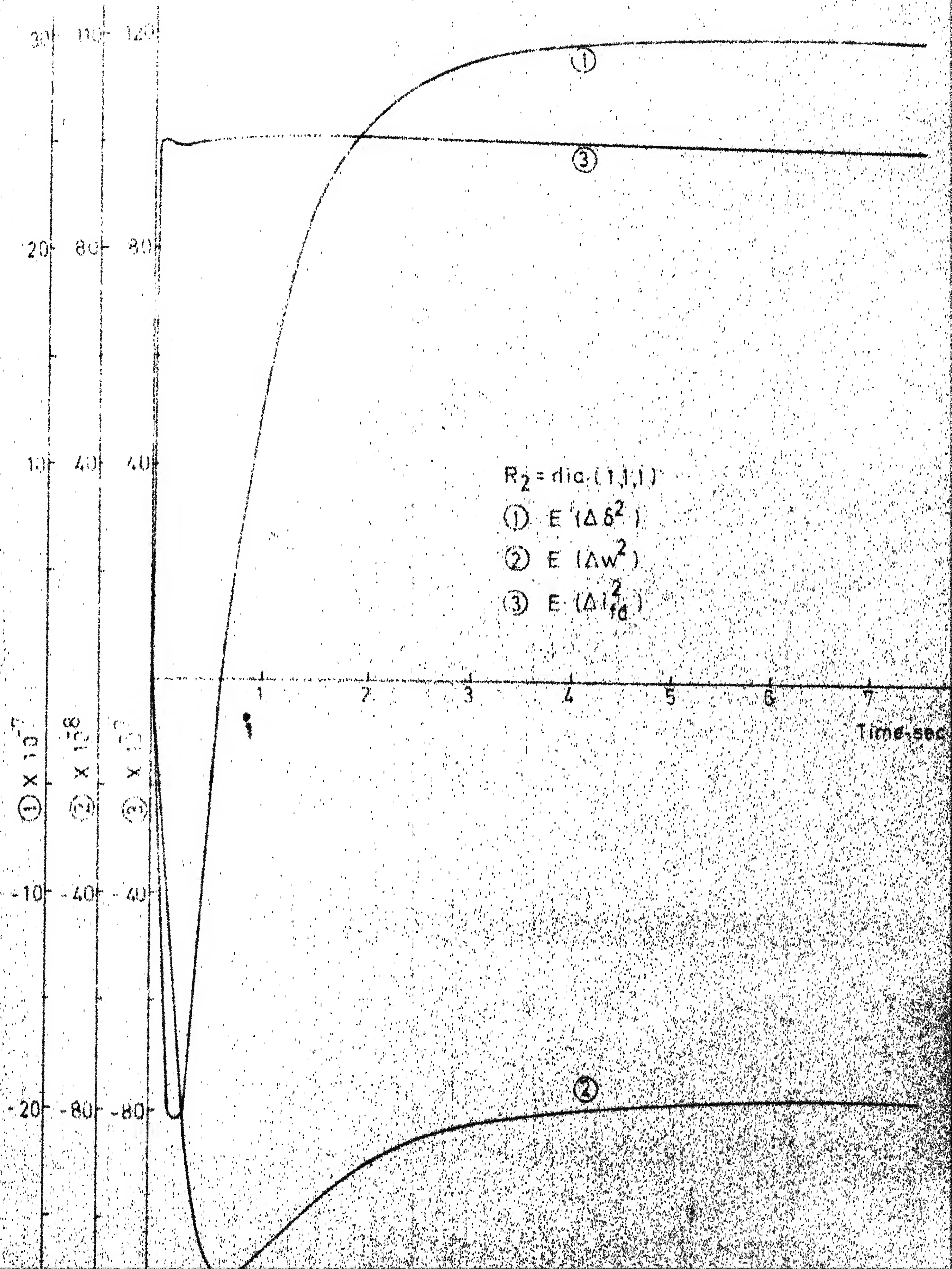
The behaviour of the optimally controlled system is then investigated in the presence of measurement errors only. It is assumed that there is no random disturbance present in the system state and/or control. Different values of the covariances of the error vector v is assumed and the increase in the value of the performance index J are obtained as given below :

R_2	ΔJ
dia(0.1 , 0.1 , 0.1)	0.1141×10^{-4}
dia(0.25, 0.25, 0.25)	0.3402×10^{-7}
dia(1.0 , 1.0 , 1.0)	0.1732×10^{-2}
dia(10.0, 10.0, 10.0)	0.1752×10^{-3}

The time variation of χ , the covariance of the state vector X given by equation(9.18) is plotted for one set of values of R_2 as shown in Figure 9.1. It is found that the increase in the value of performance index due to the presence of noise in the state and/or control is more than that due to the measurement errors.

9.4 CONCLUSION

The stochastic optimal control of synchronous machine is discussed. The optimal control law remains the same with or without the presence of noise in the



R_2 = dia (1,1,1)
① E ($\Delta\delta^2$)
② E (Δw^2)
③ E (Δi_{fd}^2)
①
②
③
① × 10^{-7}
Time-sec
1
2
3
4
5
6
7
30
20
10
-10
-20
80
40
-40
-80
120
80
40
-40
-80

state and/or output variables. However, the presence of noise increases the performance index on an average. Hence it is necessary to take into account such random disturbances while designing optimal regulators, to obtain the steady state average behaviour of the state variables. By separation theorem, the optimal regulator problem and state estimation problem are separated and solved independently. The steady state behaviour of the system is obtained in the presence of random noises. The error in the output measurements has less contribution towards the increase in the performance index than the noise term in the state.

CHAPTER X

METHOD OF SIMPLIFYING LARGE DYNAMIC SYSTEMS

10.1 INTRODUCTION

A method of simplifying large dynamic system using Schwarz canonical form is presented. This method does not require the computation of eigen values and eigen vectors as in the case of other methods; thus resulting in less computing time. The use of a reduced model obtained by state variable grouping technique to control the original system is then investigated. The performance of the original and reduced models are compared.

Large interconnected power systems demand complex computational schemes requiring excessive time and hence cost for the transient and dynamic performance analyses. The analyses may become difficult because of the limitations of computer memory and time. It is usual in control systems practice to analyse such high order systems through approximate low order systems.

10.2 STATEMENT OF THE PROBLEM

The problem posed here is that given a linear time invariant dynamic system

$$\dot{X} = A\,X + b\,u \tag{10.1}$$

it is required to find an approximate model[37]

$$\dot{X}^* = A^* X^* + b^* u \tag{10.2}$$

such that the reduced system given by above equation approximately describes the behaviour of the original system given by equation (10.1). The state variables eliminated from equation (10.1) will, therefore, have negligible effect on the system response; otherwise no simplification of the system would be possible.

10.3 SIMPLIFICATION BY SCHWARZ CANONICAL FORM

The system simplification using Schwarz canonical form requires the transformation of the original system to Schwarz form. The Schwarz canonical form is given by

$$\dot{Z} = S Z + f u \tag{10.3}$$

where S is of the form

$$S = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ -s_1 & 0 & 1 & \cdots & 0 & 0 \\ 0 & -s_2 & 0 & \cdots & 0 & 0 \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot\ \cdot\cdot & \cdot\cdot & \cdot\cdot \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & -s_{n-1} & -s_n \end{bmatrix} \tag{10.4}$$

and the vector f is given by

$$f = (0 \quad 0 \quad 0 \cdots\cdots \quad 0 \quad 1)^T \tag{10.5}$$

The transformation of any arbitrary form to Schwarz canonical form is effected by the transformation[38]

$$X = H Z \qquad (10.6)$$

In this thesis u is taken as a single variable. If u is a vector, then the transformation to Schwarz form and simplification by the present method will be rather difficult. The pair (A, b) is assumed controllable so that the transformation given by the equation (10.6) is possible. Using equations (10.1), (10.3) and (10.6), the following relations are obtained .

$$H S = A H \qquad (10.7)$$

$$H f = b \qquad (10.8)$$

If h_1, h_2, ..., h_n represent the column vectors of H, then the equations (10.7) and (10.8) can be written in terms of these column vectors as

$$(Ah_1 \vdots Ah_2 \vdots \ldots\ldots Ah_{n-1} \vdots Ah_n) = (-s_1h_2 \vdots h_1-s_2h_3 \vdots \ldots\ldots h_{n-2}-s_{n-1}h_n \vdots h_{n-1}-s_nh_n) \qquad \ldots(10.9)$$

$$h_n = b \qquad (10.10)$$

From the above two equations, the column vectors of the transformation matrix H are solved to give

$$h_n = b \qquad (10.11)$$

$$h_{n-1} = Ab + s_nb \qquad (10.12)$$

$$h_{n-k} = Ah_{n-k+1} + s_{n-k+1}\, h_{n-k+2}, \quad k = 2,\ldots,n-1 \qquad (10.13)$$

Therefore the transformation matrix H is completely determined if the elements s_i's are known. These are obtained from the following relations[38] :

$$s_i = \frac{r_{n-i+2}}{r_{n-i}} , \quad i = 1,2,\ldots,n-1 \tag{10.14}$$

$$s_n = \frac{r_2}{r_1} \tag{10.15}$$

where r_i's are the elements in the first column of the Routh matrix, determined from the characteristic equation of the system matrix A, as done for Routh stability test. Thus the transformation matrix H can be obtained easily using the equations (10.11) to (10.15).

Simplification:

The order to which the system can be simplified is obtained[43] by comparing the elements s_i in the Schwarz matrix S. The comparison starts with the ratio $\frac{s_{n-1}}{s_{n-2}}$. The comparison is said to be successful, if the ratio is greater than or equal to 10. This figure 10 has been found to be suitable for many problems[41]. It depends upon the variables of interest to be retained in the reduced model and the coefficients s_i's are related to the eigen values of the system matrix A. This figure can be determined by the physical insight into the problem. During the comparison, if the ratio $\frac{s_{n-i}}{s_{n-i-1}}$ is less than 1, the process is terminated; otherwise, it is carried on till $\frac{s_2}{s_1}$ is compared. If no comparison turns

out successful, it is inferred that no simplification would be possible. That is the eigen values are of the same order of magnitude and thus none of the variables can be neglected in the model. If ith comparison is the last successful one ($s_{n-i}/s_{n-i-1} \gtrsim 10$), then the system can be reduced to mth order where $m = n-i$.

In the reduction process, the last i-state variables are thus assumed to have negligible effect, if ith comparison is the last successful one. Then the system equation (10.3) is partitioned into m and i component vectors, giving

$$\begin{bmatrix} \dot{Z}_m \\ \cdots \\ \dot{Z}_i \end{bmatrix} = \begin{bmatrix} S_{mm} & \vdots & S_{mi} \\ \cdots & \vdots & \cdots \\ S_{im} & \vdots & S_{ii} \end{bmatrix} \begin{bmatrix} Z_m \\ \cdot\cdot \\ Z_i \end{bmatrix} + \begin{bmatrix} f_m \\ \cdot\cdot \\ f_i \end{bmatrix} u \qquad (10.16)$$

In the simplification of the systems, the last i-state variables are assumed to have less effect on the system and therefore it is assumed that

$$\dot{Z}_i = 0 \qquad (10.17)$$

Then solving for Z_i from equation (10.16)

$$Z_i = -S_{ii}^{-1}(S_{im}\, Z_m + f_i\, u) \qquad (10.18)$$

and therefore,

$$\dot{Z}_m = (S_{mm} - S_{mi}\, S_{ii}^{-1}\, S_{im})Z_m + (f_m - S_{mi}\, S_{ii}^{-1}\, f_i)u \qquad \ldots (10.19)$$

or

$$\dot{Z} = S^* Z^* + f^* u \qquad (10.20)$$

Thus the above equation gives the reduced model in the Schwarz form.

To obtain the reduced model in the original form the system equation (10.1) and the transformation equation (10.6) are again partitioned into m and l components as shown below.

$$\begin{bmatrix} \dot{X}_m \\ \cdots \\ \dot{X}_l \end{bmatrix} = \left[\begin{array}{c:c} A_{mm} & A_{ml} \\ \hdashline A_{lm} & A_{ll} \end{array}\right] \begin{bmatrix} X_m \\ \cdots \\ X_l \end{bmatrix} + \begin{bmatrix} b_m \\ \cdots \\ b_l \end{bmatrix} u \qquad (10.21)$$

$$\begin{bmatrix} X_m \\ \cdots \\ X_l \end{bmatrix} = \left[\begin{array}{c:c} H_{mm} & H_{ml} \\ \hdashline H_{lm} & H_{ll} \end{array}\right] \begin{bmatrix} Z_m \\ \cdots \\ Z_l \end{bmatrix} \qquad (10.22)$$

From the above equation, the vectors X_m and X_l are solved in terms of the vector Z_m using equation (10.18) as

$$X_m = (H_{mm} - H_{ml} S_{ll}^{-1} S_{lm})Z_m - H_{ml} S_{ll}^{-1} f_l u \qquad (10.23)$$

and

$$X_l = (H_{lm} - H_{ll} S_{ll}^{-1} S_{lm})Z_m - H_{ll} S_{ll}^{-1} f_l u \qquad (10.24)$$

Therefore from equation (10.23),

$$Z_m = H_m^* X_m + H_m^* H_{ml} S_{ll}^{-1} f_l u \qquad (10.25)$$

where

$$H_m^* = (H_{mm} - H_{mi}\, S_{ii}^{-1}\, S_{im})^{-1} \tag{10.26}$$

Hence from equation (10.24)

$$X_i = H_i^*\, H_m^*\, X_m + f_i^*\, u \tag{10.27}$$

where

$$H_i^* = H_{im} - H_{ii}\, S_{ii}^{-1}\, S_{im} \tag{10.28}$$

and

$$f_i^* = H_i^*\, H_m^*\, H_{mi}\, S_{ii}^{-1}\, f_i - H_{ii}\, S_{ii}^{-1}\, f_i \tag{10.29}$$

From equation (10.21), the dynamics of X_m are determined as

$$\dot{X}_m = A_{mm}\, X_m + A_{mi}\, X_i + b_m\, u \tag{10.30}$$

Then substituting for $\dot{X}_i$ from equation (10.27),

$$\dot{X}_m = (A_{mm} + A_{mi}\, H_i^*\, H_m^*)X_m + (A_{mi}\, f_i^* + b_m)u \tag{10.31}$$

or

$$\dot{X}^* = A^*\, X^* + b^*\, u \tag{10.32}$$

Thus the simplified model using Schwarz canonical form is obtained in the required form. The reduced system matrix A^* and the input vector b^* are given by

$$A^* = A_{mm} + A_{mi}\, H_i^*\, H_m^* \tag{10.33}$$

and

$$b^* = A_{mi}\, f_i^* + b_m \tag{10.34}$$

where H_i^*, H_m^* and f_i^* are already defined.

10.4 PERFORMANCE OF THE REDUCED SYSTEM

To illustrate the method of simplification discussed in the previous section, the response of the free system (synchronous machine infinite bus system) shown in Figure 2.1, of Chapter II is considered for impulse type distrubances. The free system matrix A is given in Chapter II for the specified operating conditions with rotor angle of 26.3°. The system is simplified by using Schwarz canonical form. The forcing function u in equations (10.3) and (10.32) is zero. The vector b is chosen as $b = (0, 0, 0, 0, 0, 0, 1)^T$ so that the pair (A, b) is controllable. The original system order is seven. It is reduced to a third order system by the present method using Schwarz canonical form. The state variables retained in the simplified model are $\Delta\delta$, Δw and $\Delta e_q'$. The reduced system matrix A^* is obtained from equation (10.33) and is given by

$$A^* = \begin{bmatrix} 0 & 1 & 0 \\ -77.72 & -1.3802 & -26.821 \\ -0.307 & 0.5725 & -0.3032 \end{bmatrix}$$

The response of the reduced system is obtained for initial disturbance in the rotor angle as shown in Figure 10.1. The original response of the rotor angle for the same disturbance is also plotted for comparison. From the two responses it is seen that the reduced model

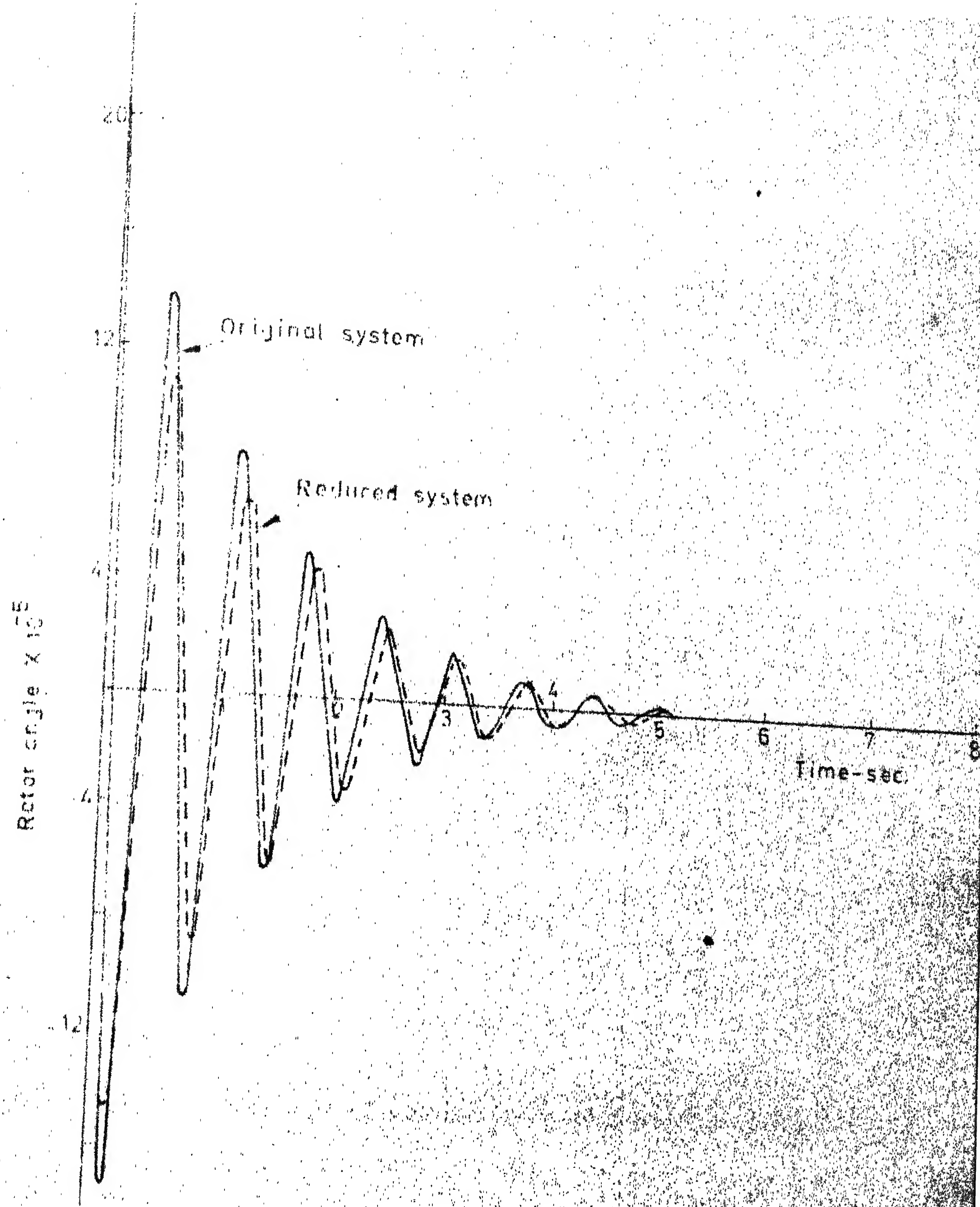


FIG. 10.1 FREE RESPONSE OF SYSTEM

by the present method has negligible errors both in the initial and final periods of the response. Thus the overall behaviour of the simplified model is the same as that of the original system.

10.5 SUBOPTIMAL CONTROL USING REDUCED MODEL

The simplified models can be used to obtain a control law for the original system which is suboptimal. An optimal regulator can be designed for the reduced model and this can be used as a suboptimal control on the original system. The simplified model can be obtained very easily for single input systems by using Schwarz canonical form as already discussed. Since it is difficult to obtain a simplified model by that method for multivariable control systems, the method of state variable grouping[40] is used for the present analysis. In this method, the state variables which have large time constants are grouped together as X_m and the rest of the variables which have negligible effect on the system with smaller time constants, are grouped together as X_1. This type of grouping is done either by having some approximate knowledge of the eigen values of the system matrix or by physical insight into the nature of the state variables. Thus partitioning the system equation

$$\dot{X} = A\,X + B\,u \tag{10.35}$$

into X_m and X_1 component vectors and assuming that

$\dot{X}_1 = 0$, the eliminated variables are solved to give

$$X_1 = -A_{11}^{-1} A_{1m} X_m - A_{11}^{-1} B_1 u \tag{10.36}$$

Then the reduced model is obtained as

$$\dot{X}_m = (A_{mm} - A_{m1} A_{11}^{-1} A_{1m})X_m + (B_m - A_{11}^{-1} B_1)u \tag{10.37}$$

or

$$\dot{X} = A_1^* X^* + B^* u \tag{10.38}$$

Thus retaining $\Delta\delta$, Δw and Δi_{fd} as the state variables for the reduced system of the original system discussed in Chapter II, the reduced system matrices A_1^* and B^* are obtained as

$$A_1^* = \begin{bmatrix} 0 & 1 & 0 \\ -79.74 & -0.0536 & -22.724 \\ -0.6098 & 0.0003 & -0.3264 \end{bmatrix}$$

$$B^* = \begin{bmatrix} -0.0031 & 0 \\ -0.05 & 52.08 \\ 2.126 & 0 \end{bmatrix}$$

This method can be used even for multivariable control systems where u can be a vector as taken in equation (10.35). It is shown[41] that the response of the reduced model obtained by this method is in close agreement with the original system response in the initial periods but deviates from the original in the steady state. The control variables used are ΔT_i and ΔE_{fd}.

Suboptimal Control:

The problem posed here is to obtain an optimal control law for the reduced system given by equation (10.38), which can be used as a suboptimal control law for the original system given by equation (10.35). The performance index for the reduced model is chosen as

$$J = \frac{1}{2} \int_0^{\infty} (X^{*T} Q^* X^* + u^T R^* u)dt \qquad (10.39)$$

The optimal control law for the reduced system is obtained as[17]

$$u = F^* X^* \qquad (10.40)$$

where

$$F^* = -R^{*-1} B^{*T} P^* \qquad (10.41)$$

P^* is obtained as a positive definite solution of the Riccati equation

$$P^* A_1^* + A_1^{*T} P^* - P^* B^* R^{*-1} B^{*T} P^* + Q^* = 0 \qquad (10.42)$$

The weightage matrices Q^* and R^* are chosen as identity matrices. The P^*-matrix for the reduced model given above is obtained as

$$P^* = \begin{bmatrix} 1.7877 & 0.0049 & 0.1035 \\ 0.0049 & 0.0193 & -0.0061 \\ 0.1035 & -0.0061 & 0.4449 \end{bmatrix}$$

Now, these three state variables namely $\Delta\delta$, Δw and Δi_{fd} are assumed to be the output variables that can

be fedback to the original system. Thus the use of this control law[39] results in a suboptimal control of the original system. The initial response of the original system by using such a suboptimal control is shown in Figure 10.2, for field current disturbance. The response is comparable with the optimal system response. It is nonoscillatory and decays fast exponentially to the steady state values. This control law is better than the conventional regulators.

10.6 CONCLUSION

The use of Schwarz canonical form in simplifying large, linear time invariant, dynamic systems is presented. This method does not require the computation of eigen values or eigen vectors of the system matrix compared to other methods[37] and hence the total computing time is much less. If the variables of interest are related to fast acting modes, good reduction would not be possible. The response of the reduced model using Schwarz canonical form is in closer agreement with the original system response. Hence this method can be used for the analysis of the dynamic behaviour of the original system by reduced models.

The problem considered in the thesis is a multivariable system; as the method of simplification by Schwarz canonical form is not easily applicable for

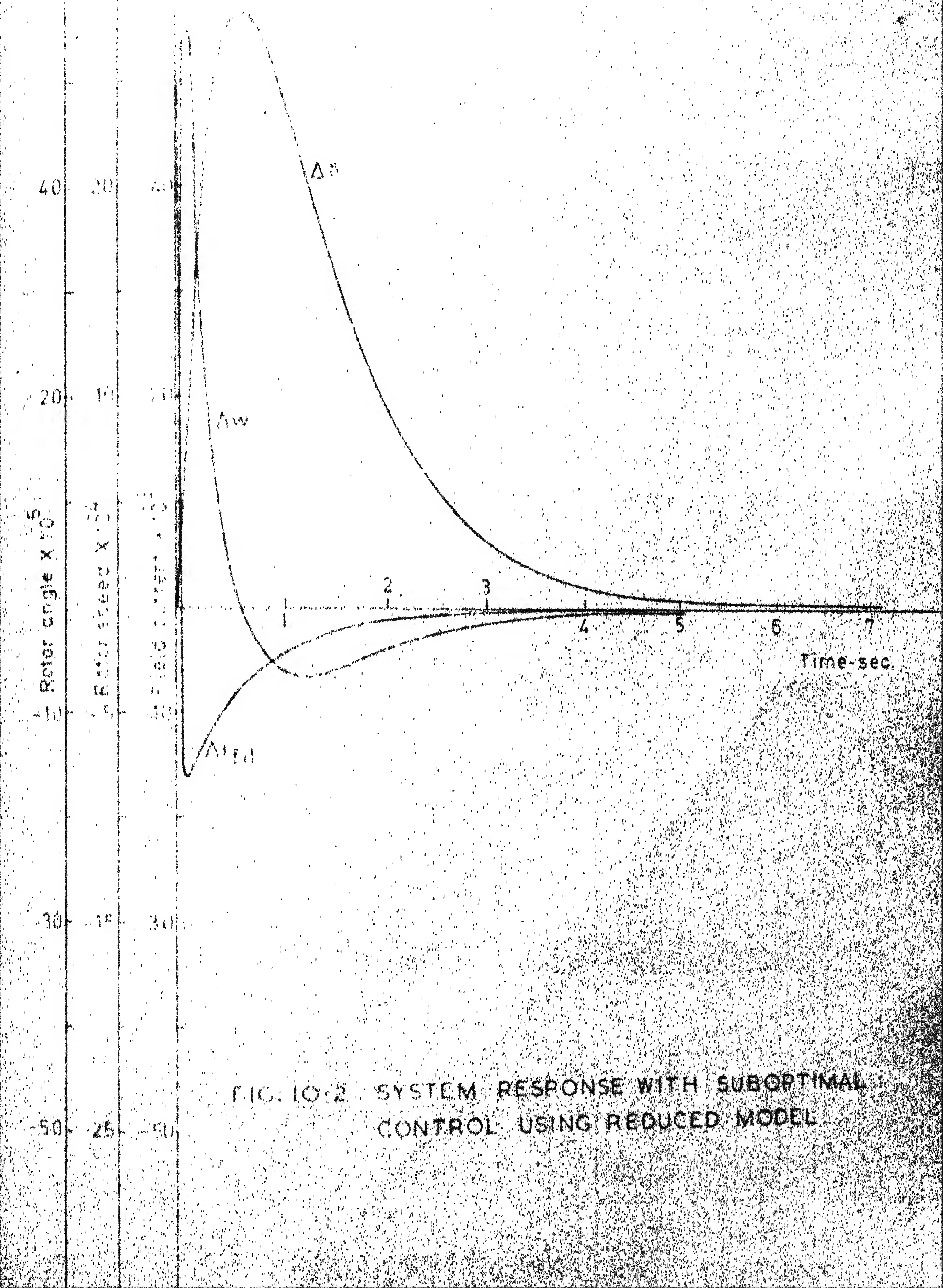


FIG. 10.2 SYSTEM RESPONSE WITH SUBOPTIMAL CONTROL USING REDUCED MODEL

such systems, the method of state variable grouping is used to obtain a control law which is optimal for the reduced system but suboptimal for the original system. The performance of the original system is investigated with this type of suboptimal control law and is found to give a reasonably good performance.

CHAPTER XI

CONCLUSION

11.1 SUMMARY AND CONCLUSIONS

The voltage and frequency control of synchronous machines by means of optimal regulators gives a better dynamic performance. The control law is derived as an integrated control which is a function of the entire state vector. The optimal feedback control law requires the measurement of deviations of the state from the post disturbance steady state conditions. The deviations are difficult to find in many practical problems. However the suboptimal control which is a function of measurable output variables only, is easy to implement and can also be used on the nonlinear models.

The efficient use of modern optimization methods depend heavily upon the state variable formulation. For the synchronous machine system considered in this thesis, a state model is derived in a most suitable form. Since the implementation of nonlinear controls is difficult a linearised state space model is obtained with the object of obtaining a linear time invariant control law. The state variables are taken as winding currents instead of flux linkages. The current variables are easier to measure than the flux linkages and also they are the

natural choice for interconnected systems wherein the system performance can be readily visualized in terms of currents rather than in terms of flux linkages.

The conventional regulator parameters have a marked effect on the system dynamic performance and therefore greater reliance is placed on well designed control systems. Existing methods (Nyquist, root locus, sensitivity method, Mitrovic method etc.) of obtaining the best values of gains and time constants in the conventional regulating equipments are discussed. The second method of Lyapunov is used to obtain these parameters which ensure stability as well as better dynamic performance. The parameters obtained by this method give a better response with minimum settling time and less overshoot.

The conventional design methods for control systems, assume apriori configurations for the regulating and stabilizing equipments. For the system considered all the state variables influence the system behaviour. Hence an optimal control law is obtained wherein the control signal is a function of all the states. Such an optimal regulator gives a superior performance compared to the conventional regulators. The control law is linear and is simple to implement provided the state variables can be measured. Also the response with this control will be nonoscillatory.

The measurement of all the state variables in a system is not always possible. A compatible dynamic observer can be employed to estimate the unavailable state variables. The observer dynamics are very much sensitive to the operating conditions and also exponentially decaying errors are introduced in the reconstructed state variables. While the optimal control law with complete state vector feedback, obtained for the linearized model about an operating point fairly remains optimal over wide load conditions, with an observer cascaded and using only the output variables to reconstruct the states, the overall control law is a function of the operating point. Hence the optimal control law with all the state variables can be used even for large disturbances. The performance of the optimally controlled system for line recolsure shows that the optimal regulators are superior in performance compared to conventional regulators. But the feedback system with dynamic observer will not be optimal when used for such large system disturbances.

The drawbacks of the optimal control system cascaded with observer is successfully overcome by means of a suboptimal control law. The control law is constrained to be a linear function of the measurable output variables. The resulting suboptimal control law gives almost an optimal performance. The performance deviation while using the particular suboptimal control law at different operating

conditions is very less. Thus the suboptimal control can be used even for large disturbances. This fact is established by investigating the performance of the system for line reclosure. The implementation of the suboptimal control law is also very simple. However it is still necessary to obtain the deviations of the measurable output variables from their post fault steady state conditions. This state has to be computed before hand which will not be physically possible. So it is necessary to consider only those output variables whose steady state values are always known like the system frequency and machine terminal voltage. The suboptimal control law with these output variables is then determined and the performance is investigated.

The synchronous machine provided with an additional field winding having suitable excitation systems, has greater stability limits even under large reactive power operations. The optimal and suboptimal control of such machines is discussed, using a linearized state model. The rotor angle of such machines is maintained constant, like the system frequency, by proper excitation of the field windings. Hence the measurement of the rotor angle for feedback is easier. Thus the suboptimal control law which is a linear function of the machine rotor angle, terminal voltage and rotor speed can be easily implemented. It is established that the system performance is better

with suboptimal regulators than with conventional angle, voltage and speed regulators. It is also found that the suboptimal control law remains same over wide range of operating conditions and hence can be used even for large perturbations.

The optimal and suboptimal controls discussed above assume that the measurement of state and output variables are accurate. However any error in the measurements will affect the performance. It is established that presence of random disturbances in the state and uncertainty in the measurements increase the value of the performance index on an average. Therefore, the design of optimal or suboptimal control should take into account such random noises. The preliminary investigation of stochastic optimal control of synchronous machines is discussed. It is found that the noise present in the system state and/or control has more contribution to the deterioration of performance than the measurement errors.

The design and analysis of multivariable control systems using the state space techniques depend largely on the use of digital computers and limitations exist regarding computer memory and time. The feasibility of analysing such systems with reduced models is investigated. The simplified synchronous machine model using Schwarz canonical form, gives a response which is in close

agreement with the original system response. This method takes less time for the reduction process because the system eigen values and eigen vectors are not required for the simplification as compared to other methods. But this method cannot be easily applied to multivariable control systems. Hence a more approximate state variable grouping technique is used to reduce the system size and to obtain an optimal regulator for the reduced system. This method is useful even for multivariable control systems. The optimal controller of reduced system thus obtained, is used as a suboptimal controller for the original system. The suboptimally controlled system response is found to be comparable with the optimal one already obtained for the complete system.

11.2 CONTRIBUTIONS

In the author's opinion, the following are the contributions made to the field of synchronous machine control using the modern and optimal control theoretic concepts. The listing also depicts the main theme of each chapter in this thesis starting from Chapter II to Chapter X.

1. An accurate state space model of single machine system is obtained in a suitable form.
2. A novel method of obtaining conventional regulator parameters using second method of Lyapunov is presented.

3. An optimal regulator is obtained which is proved to be superior to conventional regulators.
4. The difficulty in the measurement of some of the state variables is overcome by designing a compatible dynamic observer.
5. A suboptimal regulator is obtained for the system which is easy to implement on the nonlinear models as well.
6. The d.w.r. synchronous machine state model is derived so that the optimal control techniques can be applied easily for the design of control systems.
7. The optimal and suboptimal control of d.w.r. synchronous machine, which improves the dynamic performance, is obtained.
8. The preliminary investigation of stochastic optimal control of synchronous machines is presented.
9. A method of simplifying linear, time invariant, dynamic systems is proposed and solved using Schwarz canonical form.
10. The synchronous machine control is investigated with a suboptimal control, obtained for the reduced model by state variable grouping.

11.3 SCOPE FOR FURTHER RESEARCH

The problem investigated in this thesis is a single machine system. The analysis and design can be extended to multimachine systems with no difficulties, except for the increase of the system size.

The optimal conventional regulator parameters are obtained by Lyapunov's method, by using a linearised system model. The problem can be investigated for the nonlinear systems.

In the design of optimal and suboptimal regulators, it is assumed that there are no constraints either on the state/output or control variables. The problem can be studied with these constraints imposed on them. In particular the problem of magnitude constraint on the control variables resulting in bang-bang control and time optimal control problems can be investigated.

The problem of stochastic control with state dependent noise and unknown noise statistics are worth studying. The stochastic control problem with the added restriction that the control law is a function of the outputs only, will prove worthy of investigation.

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APPENDIX A

ROSENBROCK'S METHOD OF FUNCTION MINIMIZATION

The parameter optimization by Rosenbrock's hill climbing technique is discussed here. Let p be the number of parameters to be optimized and $k_1, k_2, \ldots, k_p$ be the variable parameters to be optimized. A set of p orthogonal unit vectors $\eta_1, \eta_2, \ldots, \eta_p$ and a set of p constants $e_1, e_2, \ldots, e_p$ are initially stored. The vectors η_i, $i = 1,2,\ldots,p$ are parallel to the coordinate axes of the p parameters. Initially the constants e_i's are set equal to 0.1. The criterion function J is then evaluated with k_i's set equal to zero. The value of J is evaluated once more with the parameters set equal to $K + e_1 \eta_1$, where $K = (k_1, k_2, \ldots, k_p)^T$. If the second criterion function is smaller (success), K is replaced by $K + e_1\eta_1$ and e_1 is replaced by $3e_1$: if greater (failure) K is left unaltered and e_1 is replaced by $-\frac{1}{2}e_1$. This procedure is then repeated in turn with η_1 replaced by $\eta_2, \eta_3, \ldots, \eta_p, \eta_1, \eta_2 \ldots$ until for each η_i, a success has been achieved and subsequently a failure. Then the axes are rotated in the following way.

Let the sum of all successful steps parallel to η_i be denoted by d_i. Then the following sums are obtained :

$$\begin{aligned} \alpha_1 &= d_1 \eta_1 + d_2 \eta_2 + \cdots\cdots + d_p \eta_p \\ \alpha_2 &= \qquad\quad d_2 \eta_2 + \cdots\cdots + d_p \eta_p \\ \cdot\cdot &\quad \cdot\cdot \quad \cdot\cdot \quad \cdots\cdots \quad \cdot\cdot \\ \alpha_p &= \qquad\qquad\qquad\qquad\quad d_p \eta_p \end{aligned} \tag{A.1}$$

A new set of vectors η_i is now formed by Schmidt procedure :

$$\beta_i = \alpha_i - \sum_{j=1}^{i-1} (\alpha_j^T \alpha_j)\eta_j \tag{A.2}$$

$$\eta_i = \beta_i / [\beta_i^T \beta_i] \tag{A.3}$$

for $i = 1,2,\ldots,p$.

This completes one stage of the process. With the new set of values as the starting values for the second stage, the procedure is repeated. The computations are terminated if the optimality conditions are satisfied.

APPENDIX B

CONTROLLABILITY AND OBSERVABILITY CONDITIONS

Controllability:

A system is said to be completely state controllable if it is possible to transfer any given initial state $X(t_o)$ to any desired final state $X(t_f)$ in a finite time interval $t_o \leq t \leq t_f$, with an unconstrained control vector $u(t)$.

A system is said to be completely output controllable if any given initial output $Y(t_o)$ can be transferred to the desired output $Y(t_f)$ in finite time interval $t_o \leq t \leq t_f$, with an unconstrained control vector $u(t)$.

For a linear time invariant system given by

$$\dot{X} = A X + B u \tag{B.1}$$

$$Y = C X \tag{B.2}$$

the complete state controllability implies that the composite W-matrix[15]

$$W = (B \vdots AB \vdots A^2B \vdots \ldots\ldots A^{n-1}B) \tag{B.3}$$

is of rank n.

For complete output controllability of the system given by equations (B.1) and (B.2) the composite W-matrix

$$W = (CB \vdots CAB \vdots CA^2B \vdots \ldots\ldots CA^{n-1}B) \tag{B.4}$$

is of rank m.

Observability:

A system is said to be completely observable in the time interval $t_o \leq t \leq t_f$ for any given t_o and some t_f, if every state variable can be reconstructed from the knowledge of the output variables Y(t) and the control vector u(t) in the time interval $t_o \leq t \leq t_f$.

The system given by equations (B.1) and (B.2) is completely observable if the composite W-matrix

$$W = [C^T \vdots A^T C^T \vdots (A^T)^2 C^T \vdots \ldots\ldots (A^T)^{n-1} C^T] \quad \text{(B.5)}$$

is of rank n.

The existence of optimal control law requires the prior investigation of the system state and/or output controllability. For the design of an observer system, the notion of observability is necessary. These conditions are invoked before a solution to the problem is attempted.

APPENDIX C

OPTIMAL LINEAR REGULATOR

The linear feedback control law is derived using Pontryagin's minimum principle for the system

$$\dot{X} = A\,X + B\,u,\; X(t_o) = X_o \tag{C.1}$$

It is required to find the control which minimizes the cost function

$$J = \frac{1}{2} X^T S\, X\Big|_{t=t_f} + \frac{1}{2}\int_o^{t_f} (X^T Q\, X + u^T R\, u)dt \tag{C.2}$$

Without loss of generality it is assumed that the Q, R and S are symmetric matrices. The Hamiltonian is formed as

$$H(X, u, \lambda, t) = \frac{1}{2} X^T Q\, X + \frac{1}{2} u^T R\, u + \lambda^T(A\,X + B\,u) \quad \ldots \tag{C.3}$$

where λ is the Lagrangian multiplier vector. Application of the minimum principle requires that for an optimal control,

$$\frac{\partial H}{\partial u} = 0 = R\,u + B^T\lambda \tag{C.4}$$

and

$$\frac{\partial H}{\partial X} = -\dot{\lambda} = Q\,X + A^T\lambda \tag{C.5}$$

with the terminal condition that

$$\lambda(t_f) = S\,X(t_f) \tag{C.6}$$

Thus the optimal control is given by

$$u = - R^{-1} B^T \lambda \tag{C.7}$$

A closed loop optimal control law is obtained by assuming a solution to equation (C.5) in a form similar to the equation (C.6). Thus let

$$\lambda = P X \tag{C.8}$$

be a solution.Substituting equation (C.8) in equations (C.1) and (C.7), the following equation is obtained :

$$\dot{X} = A X - B R^{-1} B^T P X \tag{C.9}$$

and from equations (C.5) and (C.8)

$$\dot{\lambda} = \dot{P} X + P \dot{X} = - Q X - A^T P X \tag{C.10}$$

Combining equations (C.9) and (C.10),

$$(\dot{P} + P A + A^T P - P B R^{-1} B^T P + Q) X = 0 \tag{C.11}$$

Since this must hold good for all nonzero X, the term premultiplying X must be zero. Thus the symmetric P-matrix must satisfy the Riccati differential equation

$$\dot{P} = - P A - A^T P + P B R^{-1} B^T P - Q \tag{C.12}$$

with terminal condition,

$$P = S \quad \text{at} \quad t = t_f \tag{C.13}$$

Thus a closed loop optimal control law is obtained as

$$u = -R^{-1} B^T P X \tag{C.15}$$

For linear time invariant systems with constant Q and R matrices and for the infinite time regulator the

Riccati differential equation (C.12) reduces to an algebraic equation

$$PA + A^T P - P B R^{-1} B^T P + Q = 0 \qquad (C.16)$$

Then the control law becomes a constant feedback of the state vector.

APPENDIX D

RUNGE-KUTTA METHOD

The Runge-Kutta fourth order integration method is described for the solution of vector differential equation. Consider the system

$$\dot{X} = f(X, t), \quad X(o) = X_o \tag{D.1}$$

If the value of X is known at time t, then the value of the solution vector can be obtained at time $t + \Delta t$ as

$$X(t + \Delta t) = X(t) + (K_1 + 2K_2 + 2K_3 + K_4)/6 \tag{D.2}$$

where the vectors K_1, K_2, K_3 and K_4 are calculated from

$$K_1 = \Delta t\, f(X,t) \tag{D.3}$$

$$K_2 = \Delta t\, f[(X + \frac{K_1}{2}), (t + \frac{t}{2})] \tag{D.4}$$

$$K_3 = \Delta t\, f[(X + \frac{K_2}{2}), (t + \frac{t}{2})] \tag{D.5}$$

$$K_4 = \Delta t\, f[(X + K_3), (t + \Delta t)] \tag{D.6}$$

The equation (D.2) is obtained by using the truncated Taylor series expansion of the function f(X,t) about the starting point X(t) and equating the corresponding coefficients of $(\Delta t)^n$, $n = 1,2,3,4$. The method has a rounding of errors proportional to $(\Delta t)^5$. The method is numerically stable for reasonable values of Δt.

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2. M. Arumugam and M. Ramamoorty, "Optimal Selection of Controller Parameters using Second Method of Lyapunov", Electronics Letters, Vol.7, No.13, 1st July, 1971, pp. 365-367.
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4. M. Ramamoorty and M. Agumugam, "Design of Constant Output Feedback Controllers for a Synchronous Machine", Paper accepted for Publication in Proc. IEE, London.

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